# Package 'uGMAR'

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**Title** Estimate Univariate Gaussian and Student's t Mixture Autoregressive Models

Version 3.6.0

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**Description** Maximum likelihood estimation of univariate Gaussian Mixture Autoregressive (GMAR), Student's t Mixture Autoregressive (StMAR), and Gaussian and Student's t Mixture Autoregressive (G-StMAR) models,

quantile residual tests, graphical diagnostics, forecast and simulate from GMAR, StMAR and G-StMAR processes.

Leena Kalliovirta, Mika Meitz, Pentti Saikkonen (2015) <doi:10.1111/jtsa.12108>, Mika Meitz, Daniel Preve, Pentti Saikkonen (2023) <doi:10.1080/03610926.2021.1916531>, Savi Virolainen (2022) <doi:10.1515/snde-2020-0060>.

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BugReports https://github.com/saviviro/uGMAR/issues

License GPL-3

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uGMAR-package

uGMAR: Estimate Univariate Gaussian and Student's t Mixture Autoregressive Models

## **Description**

uGMAR is a package for estimating univariate Gaussian mixture autoregressive (GMAR), Student's t mixture autoregressive (StMAR), and Gaussian and Student's t mixture autoregressive (G-StMAR) models. In addition to unconstrained and constrained estimation, uGMAR provides tools for quantile residual based model diagnostics, forecasting, simulation, and more.

The readme file or the vignette is a good place to start.

## Author(s)

Maintainer: Savi Virolainen <savi.virolainen@helsinki.fi>(ORCID)

#### See Also

Useful links:

• Report bugs at https://github.com/saviviro/uGMAR/issues

add\_data

Add data to object of class 'gsmar' defining a GMAR, StMAR, or G-StMAR model

# **Description**

add\_data adds or updates data to object of class 'gsmar' that defines a GMAR, StMAR, or G-StMAR model. Also calculates empirical mixing weights, conditional moments, and quantile residuals accordingly.

# Usage

```
add_data(
   data,
   gsmar,
   calc_qresiduals = TRUE,
   calc_cond_moments = TRUE,
   calc_std_errors = FALSE,
   custom_h = NULL
)
```

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## Arguments

data a numeric vector or class 'ts' object containing the data. NA values are not

supported.

gsmar a class 'gsmar' object, typically generated by fitGSMAR or GSMAR.

calc\_qresiduals

should quantile residuals be calculated? Default is TRUE iff the model contains

data.

calc\_cond\_moments

should conditional means and variances be calculated? Default is TRUE iff the

model contains data.

calc\_std\_errors

should approximate standard errors be calculated?

custom\_h

A numeric vector with same the length as the parameter vector: i:th element of custom\_h is the difference used in central difference approximation for partial differentials of the log-likelihood function for the i:th parameter. If NULL (default), then the difference used for differentiating overly large degrees of freedom parameters is adjusted to avoid numerical problems, and the difference is

6e-6 for the other parameters.

#### Value

Returns an object of class 'gsmar' defining the GMAR, StMAR, or G-StMAR model with the data added to the model. If the object already contained data, the data will be updated. Does not modify the 'gsmar' object given as argument!

# References

- Kalliovirta L., Meitz M. and Saikkonen P. 2015. Gaussian Mixture Autoregressive model for univariate time series. *Journal of Time Series Analysis*, **36**(2), 247-266.
- Meitz M., Preve D., Saikkonen P. 2023. A mixture autoregressive model based on Student's t-distribution. *Communications in Statistics Theory and Methods*, **52**(2), 499-515.
- Virolainen S. 2022. A mixture autoregressive model based on Gaussian and Student's t-distributions. Studies in Nonlinear Dynamics & Econometrics, 26(4) 559-580.

#### See Also

```
\label{lem:condition} fit GSMAR, GSMAR, iterate\_more, get\_gradient, get\_regime\_means, swap\_parametrization, stmar\_to\_gstmar
```

## **Examples**

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```
gstmar42 <- add_data(data=M10Y1Y, gsmar=gstmar42)
gstmar42</pre>
```

alt\_gsmar

Construct a GSMAR model based on results from an arbitrary estimation round of fitGSMAR

## **Description**

alt\_gsmar constructs a GSMAR model based on results from an arbitrary estimation round of fitGSMAR.

# Usage

```
alt_gsmar(
   gsmar,
   which_round = 1,
   which_largest,
   calc_qresiduals = TRUE,
   calc_cond_moments = TRUE,
   calc_std_errors = TRUE,
   custom_h = NULL
)
```

# **Arguments**

gsmar a class 'gsmar' object, typically generated by fitGSMAR or GSMAR.

which\_round based on which estimation round should the model be constructed? An integer

value in 1,...,ncalls.

which\_largest based on estimation round with which largest log-likelihood should the model be

constructed? An integer value in 1,...,ncalls. For example, which\_largest=2 would take the second largest log-likelihood and construct the model based on

the corresponding estimates. If specified, then which\_round is ignored.

calc\_qresiduals

should quantile residuals be calculated? Default is TRUE iff the model contains

calc\_cond\_moments

should conditional means and variances be calculated? Default is TRUE iff the model contains data.

calc\_std\_errors

should approximate standard errors be calculated?

custom\_h

A numeric vector with same the length as the parameter vector: i:th element of custom\_h is the difference used in central difference approximation for partial differentials of the log-likelihood function for the i:th parameter. If NULL (default), then the difference used for differentiating overly large degrees of freedom parameters is adjusted to avoid numerical problems, and the difference is 6e-6 for the other parameters.

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#### **Details**

It's sometimes useful to examine other estimates than the one with the highest log-likelihood value. This function is just a simple wrapper to GSMAR that picks the correct estimates from an object returned by fitGSMAR.

In addition to the S3 methods listed under the topic "Methods (by generic)", the predict and simulate methods are also available for the class 'gsmar' objects (see ?predict.gsmar and ?simulate.gsmar).

#### Value

Returns an object of class 'gsmar' defining the specified GMAR, StMAR, or G-StMAR model. If data is supplied, the returned object contains (by default) empirical mixing weights, some conditional and unconditional moments, and quantile residuals. Note that the first p observations are taken as the initial values so the mixing weights, conditional moments, and quantile residuals start from the p+1:th observation (interpreted as t=1).

#### References

- Kalliovirta L., Meitz M. and Saikkonen P. 2015. Gaussian Mixture Autoregressive model for univariate time series. *Journal of Time Series Analysis*, **36**(2), 247-266.
- Meitz M., Preve D., Saikkonen P. 2023. A mixture autoregressive model based on Student's t-distribution. *Communications in Statistics Theory and Methods*, **52**(2), 499-515.
- Virolainen S. 2022. A mixture autoregressive model based on Gaussian and Student's t-distributions. Studies in Nonlinear Dynamics & Econometrics, **26**(4) 559-580.

## See Also

```
fitGSMAR, GSMAR, iterate_more, get_gradient, get_regime_means, swap_parametrization,
stmar_to_gstmar
```

#### **Examples**

calc\_gradient 7

gradient or Hessian matrix
e

# **Description**

calc\_gradient or calc\_hessian calculates the gradient or Hessian matrix of the given function at the given point using central difference numerical approximation. get\_gradient (and get\_foc) or get\_hessian calculates the gradient or Hessian matrix of the log-likelihood function at the parameter values of a class 'gsmar' object. get\_soc returns eigenvalues of the Hessian matrix.

# Usage

```
calc_gradient(x, fn, h = 6e-06, varying_h = NULL, ...)
calc_hessian(x, fn, h = 6e-06, varying_h = NULL, ...)
get_gradient(gsmar, custom_h = NULL)
get_foc(gsmar, custom_h = NULL)
get_hessian(gsmar, custom_h = NULL)
get_soc(gsmar, custom_h = NULL)
```

## **Arguments**

х	a numeric vector specifying the point at which the gradient or Hessian should be evaluated.
fn	a function that takes in the argument x as the <b>first</b> argument.
h	the difference used to approximate the derivatives.
varying_h	a numeric vector with the same length as $x$ specifying the difference $h$ for each dimension separately. If NULL (default), then the difference given as parameter $h$ will be used for all dimensions.
	other arguments passed to fn.
gsmar	a class 'gsmar' object, typically generated by fitGSMAR or GSMAR.
custom_h	same as varying_h but if NULL (default), then the difference h used for differentiating overly large degrees of freedom parameters is adjusted to avoid numerical problems, and the difference is 6e-6 for the other parameters.

#### **Details**

In particular, the functions get\_foc and get\_soc can be used to check whether the found estimates denote a (local) maximum point, a saddle point, or something else.

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## Value

The gradient functions return numerical approximation of the gradient, and the Hessian functions return numerical approximation of the Hessian. get\_soc returns eigenvalues of the Hessian matrix, get\_foc is the same as get\_gradient but named conveniently.

## Warning

No argument checks!

#### See Also

```
profile_logliks
```

## **Examples**

```
# Simple function
foo <- function(x) x^2 + x
calc_gradient(x=1, fn=foo)
calc_gradient(x=-0.5, fn=foo)
calc_hessian(x=2, fn=foo)
# More complicated function
foo <- function(x, a, b) a*x[1]^2 - b*x[2]^2
calc_gradient(x=c(1, 2), fn=foo, a=0.3, b=0.1)
calc_hessian(x=c(1, 2), fn=foo, a=0.3, b=0.1)
# GMAR model
params12 <- c(1.70, 0.85, 0.30, 4.12, 0.73, 1.98, 0.63)
gmar12 <- GSMAR(data=simudata, p=1, M=2, params=params12, model="GMAR")</pre>
get_gradient(gmar12)
get_foc(gmar12)
get_hessian(gmar12)
get_soc(gmar12)
```

 $\verb|cond_moments||$ 

Calculate conditional moments of GMAR, StMAR, or G-StMAR model

## Description

cond\_moments calculates the regime specific conditional means and variances and total conditional means and variances of the specified GMAR, StMAR or G-StMAR model.

## Usage

```
cond_moments(
  data,
  p,
  M,
```

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```
params,
model = c("GMAR", "StMAR", "G-StMAR"),
restricted = FALSE,
constraints = NULL,
parametrization = c("intercept", "mean"),
to_return = c("regime_cmeans", "regime_cvars", "total_cmeans", "total_cvars")
```

#### **Arguments**

data

a numeric vector or class 'ts' object containing the data. NA values are not supported.

р

a positive integer specifying the autoregressive order of the model.

М

**For GMAR and StMAR models:** a positive integer specifying the number of mixture components.

For G-StMAR models: a size  $(2 \times 1)$  integer vector specifying the number of *GMAR type* components M1 in the first element and *StMAR type* components M2 in the second element. The total number of mixture components is M=M1+M2.

params

a real valued parameter vector specifying the model.

For non-restricted models: Size  $(M(p+3)+M-M1-1\times 1)$  vector  $\theta=(v_1,...,v_M,$ 

 $\alpha_1, ..., \alpha_{M-1}, \nu$ ) where

- $v_m = (\phi_{m,0}, \phi_m, \sigma_m^2)$
- $\phi_m = (\phi_{m,1}, ..., \phi_{m,p}), m = 1, ..., M$
- $\nu = (\nu_{M1+1}, ..., \nu_M)$
- M1 is the number of GMAR type regimes.

In the GMAR model, M1=M and the parameter  $\nu$  dropped. In the StMAR model, M1=0.

If the model imposes **linear constraints** on the autoregressive parameters: Replace the vectors  $\phi_m$  with the vectors  $\psi_m$  that satisfy  $\phi_m = C_m \psi_m$  (see the argument constraints).

For restricted models: Size  $(3M+M-M1+p-1\times1)$  vector  $\theta=(\phi_{1,0},...,\phi_{M,0},\phi,\sigma_1^2,...,\sigma_M^2,\alpha_1,...,\alpha_{M-1},\nu)$ , where  $\phi=(\phi_1,...,\phi_p)$  contains the AR coefficients, which are common for all regimes.

If the model imposes **linear constraints** on the autoregressive parameters: Replace the vector  $\phi$  with the vector  $\psi$  that satisfies  $\phi$ = $C\psi$  (see the argument constraints).

Symbol  $\phi$  denotes an AR coefficient,  $\sigma^2$  a variance,  $\alpha$  a mixing weight, and  $\nu$  a degrees of freedom parameter. If parametrization=="mean", just replace each intercept term  $\phi_{m,0}$  with the regimewise mean  $\mu_m = \phi_{m,0}/(1-\sum\phi_{i,m})$ . In the **G-StMAR** model, the first M1 components are *GMAR type* and the rest M2 components are *StMAR type*. Note that in the case **M=1**, the mixing weight parameters  $\alpha$  are dropped, and in the case of **StMAR** or **G-StMAR** model, the degrees of freedom parameters  $\nu$  have to be larger than 2.

model

is "GMAR", "StMAR", or "G-StMAR" model considered? In the G-StMAR model, the first M1 components are *GMAR type* and the rest M2 components are *StMAR type*.

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restricted

a logical argument stating whether the AR coefficients  $\phi_{m,1},...,\phi_{m,p}$  are restricted to be the same for all regimes.

constraints

specifies linear constraints imposed to each regime's autoregressive parameters separately.

For non-restricted models: a list of size  $(p \times q_m)$  constraint matrices  $C_m$  of full column rank satisfying  $\phi_m = C_m \psi_m$  for all m = 1, ..., M, where  $\phi_m = (\phi_{m,1}, ..., \phi_{m,p})$  and  $\psi_m = (\psi_{m,1}, ..., \psi_{m,q_m})$ .

For restricted models: a size  $(p \times q)$  constraint matrix C of full column rank satisfying  $\phi = C\psi$ , where  $\phi = (\phi_1, ..., \phi_p)$  and  $\psi = \psi_1, ..., \psi_q$ .

The symbol  $\phi$  denotes an AR coefficient. Note that regardless of any constraints, the autoregressive order is always p for all regimes. Ignore or set to NULL if applying linear constraints is not desired.

## parametrization

is the model parametrized with the "intercepts"  $\phi_{m,0}$  or "means"  $\mu_m = \phi_{m,0}/(1-\sum \phi_{i,m})$ ?

to\_return

calculate regimewise conditional means (regime\_cmeans), regimewise conditional variances (regime\_cvars), total conditional means (total\_cmeans), or total conditional variances (total\_cvars)?

## Value

Note that the first p observations are taken as the initial values so the conditional moments start form the p+1:th observation (interpreted as t=1).

if to\_return=="regime\_cmeans": a size  $((n_obs-p)\times M)$  matrix containing the regime specific conditional means.

if to\_return=="regime\_cvars": a size  $((n_obs-p)\times M)$  matrix containing the regime specific conditional variances.

if to\_return=="total\_cmeans": a size  $((n_obs-p)\times 1)$  vector containing the total conditional means.

if to\_return=="total\_cvars": a size  $((n_obs-p)\times 1)$  vector containing the total conditional variances.

#### References

- Galbraith, R., Galbraith, J. 1974. On the inverses of some patterned matrices arising in the theory of stationary time series. *Journal of Applied Probability* **11**, 63-71.
- Kalliovirta L. (2012) Misspecification tests based on quantile residuals. *The Econometrics Journal*, **15**, 358-393.
- Kalliovirta L., Meitz M. and Saikkonen P. 2015. Gaussian Mixture Autoregressive model for univariate time series. *Journal of Time Series Analysis*, 36(2), 247-266.
- Meitz M., Preve D., Saikkonen P. 2023. A mixture autoregressive model based on Student's t-distribution. *Communications in Statistics Theory and Methods*, **52**(2), 499-515.
- Virolainen S. 2022. A mixture autoregressive model based on Gaussian and Student's t-distributions. Studies in Nonlinear Dynamics & Econometrics, 26(4) 559-580.

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#### See Also

```
Other moment functions: get_regime_autocovs(), get_regime_means(), get_regime_vars(), uncond_moments()
```

#### **Examples**

cond\_moment\_plot

Conditional mean or variance plot for GMAR, StMAR, and G-StMAR models

# **Description**

cond\_moment\_plot plots the one-step in-sample conditional means/variances of the model along with the time series contained in the model (e.g. the time series the model was fitted to). Also plots the regimewise conditional means/variances multiplied with the mixing weights.

# Usage

```
cond_moment_plot(gsmar, which_moment = c("mean", "variance"))
```

# Arguments

```
gsmar a class 'gsmar' object, typically generated by fitGSMAR or GSMAR. which_moment should conditional means or variances be plotted?
```

## **Details**

The conditional mean plot works best if the data contains positive values only.

#### Value

cond\_moment\_plot only plots to a graphical device and does not return anything. Numerical values of the conditional means/variances can be extracted from the model with the dollar sign.

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#### References

- Galbraith, R., Galbraith, J. 1974. On the inverses of some patterned matrices arising in the theory of stationary time series. *Journal of Applied Probability* **11**, 63-71.
- Kalliovirta L. (2012) Misspecification tests based on quantile residuals. *The Econometrics Journal*, **15**, 358-393.
- Kalliovirta L., Meitz M. and Saikkonen P. 2015. Gaussian Mixture Autoregressive model for univariate time series. *Journal of Time Series Analysis*, **36**(2), 247-266.
- Meitz M., Preve D., Saikkonen P. 2023. A mixture autoregressive model based on Student's t-distribution. *Communications in Statistics Theory and Methods*, **52**(2), 499-515.
- Virolainen S. 2022. A mixture autoregressive model based on Gaussian and Student's t-distributions. Studies in Nonlinear Dynamics & Econometrics, 26(4) 559-580.

#### See Also

profile\_logliks, diagnostic\_plot, fitGSMAR, GSMAR, quantile\_residual\_tests, quantile\_residual\_plot

## **Examples**

diagnostic\_plot

Quantile residual based diagnostic plots for GMAR, StMAR, and G-StMAR models

# Description

diagnostic\_plot plots quantile residual time series, normal QQ-plot, autocorrelation function, and squared quantile residual autocorrelation function. There is an option to also plot the individual statistics associated with the quantile residual tests (for autocorrelation and conditional heteroskedasticity) divided by their approximate standard errors with their approximate 95% critical bounds (see Kalliovirta 2012, Section 3).

## Usage

```
diagnostic_plot(gsmar, nlags = 20, nsimu = 1, plot_indstats = FALSE)
```

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## Arguments

gsmar a class 'gsmar' object, typically generated by fitGSMAR or GSMAR.

nlags a positive integer specifying how many lags should be calculated for the auto-

correlation and conditional heteroscedasticity statistics.

nsimu a positive integer specifying to how many simulated values from the process

the covariance matrix "Omega" (used to compute the tests) should be based on. Larger number of simulations may result more reliable tests but takes longer to compute. If smaller than data size, then "Omega" will be based on the given

data. Ignored if plot\_indstats==FALSE.

plot\_indstats set TRUE if the individual statistics discussed in Kalliovirta (2012) should be

plotted with their approximate 95% critical bounds (this may take some time).

#### **Details**

Sometimes the individual statistics are not plotted because it was not (numerically) possible to calculate all the required statistics. This may suggest that the model is misspecified.

The dashed lines plotted with autocorrelation functions (for quantile residuals and their squares) are plus-minus  $1.96*T^{-1/2}$  where T is the sample size (minus the p initial values for conditional models).

#### Value

diagnostic\_plot only plots to a graphical device and does not return anything. Use the function quantile\_residual\_tests in order to obtain the individual statistics.

## Suggested packages

Install the suggested package "gsl" for faster evaluations in the cases of StMAR and G-StMAR models. For large StMAR and G-StMAR models with large data the calculations to obtain the individual statistics may take a significantly long time without the package "gsl".

# References

- Galbraith, R., Galbraith, J. 1974. On the inverses of some patterned matrices arising in the theory of stationary time series. *Journal of Applied Probability* **11**, 63-71.
- Kalliovirta L. (2012) Misspecification tests based on quantile residuals. The Econometrics Journal, 15, 358-393.
- Kalliovirta L., Meitz M. and Saikkonen P. 2015. Gaussian Mixture Autoregressive model for univariate time series. *Journal of Time Series Analysis*, 36(2), 247-266.
- Meitz M., Preve D., Saikkonen P. 2023. A mixture autoregressive model based on Student's t-distribution. *Communications in Statistics Theory and Methods*, **52**(2), 499-515.
- Virolainen S. 2022. A mixture autoregressive model based on Gaussian and Student's t-distributions. Studies in Nonlinear Dynamics & Econometrics, 26(4) 559-580.

#### See Also

profile\_logliks, get\_foc, fitGSMAR, cond\_moment\_plot, quantile\_residual\_tests, quantile\_residual\_plot, simulate.gsmar, LR\_test, Wald\_test

## **Examples**

fitGSMAR

Estimate Gaussian or Student's t Mixture Autoregressive model

## Description

fitGSMAR estimates GMAR, StMAR, or G-StMAR model in two phases. In the first phase, a genetic algorithm is employed to find starting values for a gradient based method. In the second phase, the gradient based variable metric algorithm is utilized to accurately converge to a local maximum or a saddle point near each starting value. Parallel computing is used to conduct multiple rounds of estimations in parallel.

## Usage

```
fitGSMAR(
  data,
  p,
  M,
  model = c("GMAR", "StMAR", "G-StMAR"),
  restricted = FALSE,
  constraints = NULL,
  conditional = TRUE,
  parametrization = c("intercept", "mean"),
  ncalls = round(10 + 9 * log(sum(M))),
  ncores = 2,
  maxit = 500,
  seeds = NULL,
  print_res = TRUE,
  filter_estimates = TRUE,
```

)

#### **Arguments**

М

data a numeric vector or class 'ts' object containing the data. NA values are not

supported.

a positive integer specifying the autoregressive order of the model. p

For GMAR and StMAR models: a positive integer specifying the number of mixture components.

> For G-StMAR models: a size  $(2 \times 1)$  integer vector specifying the number of GMAR type components M1 in the first element and StMAR type components M2 in the second element. The total number of mixture components is M=M1+M2.

> is "GMAR", "StMAR", or "G-StMAR" model considered? In the G-StMAR model, the first M1 components are GMAR type and the rest M2 components are StMAR type.

a logical argument stating whether the AR coefficients  $\phi_{m,1},...,\phi_{m,p}$  are restricted to be the same for all regimes.

specifies linear constraints imposed to each regime's autoregressive parameters separately.

For non-restricted models: a list of size  $(p \times q_m)$  constraint matrices  $C_m$  of full column rank satisfying  $\phi_m = C_m \psi_m$  for all m = 1, ..., M, where  $\phi_m =$  $(\phi_{m,1},...,\phi_{m,p})$  and  $\psi_m = (\psi_{m,1},...,\psi_{m,q_m}).$ 

For restricted models: a size  $(p \times q)$  constraint matrix C of full column rank satisfying  $\phi = C\psi$ , where  $\phi = (\phi_1, ..., \phi_p)$  and  $\psi = \psi_1, ..., \psi_q$ .

The symbol  $\phi$  denotes an AR coefficient. Note that regardless of any constraints, the autoregressive order is always p for all regimes. Ignore or set to NULL if applying linear constraints is not desired.

a logical argument specifying whether the conditional or exact log-likelihood function should be used.

parametrization

is the model parametrized with the "intercepts"  $\phi_{m,0}$  or "means"  $\mu_m = \phi_{m,0}/(1-$ 

a positive integer specifying how many rounds of estimation should be conducted. The estimation results may vary from round to round because of multimodality of the log-likelihood function and the randomness associated with the genetic algorithm.

the number of CPU cores to be used in the estimation process. ncores

the maximum number of iterations for the variable metric algorithm.

seeds a length ncalls vector containing the random number generator seed for each call to the genetic algorithm, or NULL for not initializing the seed. Exists for the

purpose of creating reproducible results.

should the estimation results be printed? print\_res

model

restricted

constraints

conditional

ncalls

maxit

filter\_estimates

should likely inappropriate estimates be filtered? See details.

... additional settings passed to the function GAfit employing the genetic algorithm.

#### **Details**

Because of complexity and multimodality of the log-likelihood function, it's **not guaranteed** that the estimation algorithm will end up in the global maximum point. It's often expected that most of the estimation rounds will end up in some local maximum point instead, and therefore a number of estimation rounds is required for reliable results. Because of the nature of the models, the estimation may fail particularly in the cases where the number of mixture components is chosen too large. Note that the genetic algorithm is designed to avoid solutions with mixing weights of some regimes too close to zero at almost all times ("redundant regimes") but the settings can, however, be adjusted (see 'GAfit).

If the iteration limit for the variable metric algorithm (maxit) is reached, one can continue the estimation by iterating more with the function iterate\_more.

The core of the genetic algorithm is mostly based on the description by *Dorsey and Mayer* (1995). It utilizes a slightly modified version the individually adaptive crossover and mutation rates described by *Patnaik and Srinivas* (1994) and employs (50%) fitness inheritance discussed by *Smith*, *Dike and Stegmann* (1995). Large (in absolute value) but stationary AR parameter values are generated with the algorithm proposed by Monahan (1984).

The variable metric algorithm (or quasi-Newton method, Nash (1990, algorithm 21)) used in the second phase is implemented with function the optim from the package stats.

#### Additional Notes about the estimates:

Sometimes the found MLE is very close to the boundary of the stationarity region some regime, the related variance parameter is very small, and the associated mixing weights are "spiky". This kind of estimates often maximize the log-likelihood function for a technical reason that induces by the endogenously determined mixing weights. In such cases, it might be more appropriate to consider the next-best local maximum point of the log-likelihood function that is well inside the parameter space. Models based local-only maximum points can be built with the function alt\_gsmar by adjusting the argument which\_largest accordingly.

Some mixture components of the StMAR model may sometimes get very large estimates for the degrees of freedom parameters. Such parameters are weakly identified and induce various numerical problems. However, mixture components with large degree of freedom parameter estimates are similar to the mixture components of the GMAR model. It's hence advisable to further estimate a G-StMAR model by allowing the mixture components with large degrees of freedom parameter estimates to be GMAR type with the function stmar\_to\_gstmar.

Filtering inappropriate estimates: If filter\_estimates == TRUE, the function will automatically filter out estimates that it deems "inappropriate". That is, estimates that are not likely solutions of interest. Specifically, it filters out solutions that incorporate regimes with any modulus of the roots of the AR polynomial less than 1.0015; a variance parameter estimat near zero (less than 0.0015); mixing weights such that they are close to zero for almost all t for at least one regime; or mixing weight parameter estimate close to zero (or one). You can also set filter\_estimates=FALSE and find the solutions of interest yourself by using the function alt\_gsmar.

#### Value

Returns an object of class 'gsmar' defining the estimated GMAR, StMAR or G-StMAR model. The returned object contains estimated mixing weights, some conditional and unconditional moments, and quantile residuals. Note that the first p observations are taken as the initial values, so the mixing weights, conditional moments, and quantile residuals start from the p+1:th observation (interpreted as t=1). In addition, the returned object contains the estimates and log-likelihoods from all of the estimation rounds. See ?GSMAR for the form of the parameter vector, if needed.

#### S3 methods

The following S3 methods are supported for class 'gsmar' objects: print, summary, plot, predict, simulate, logLik, residuals.

#### References

- Dorsey R. E. and Mayer W. J. 1995. Genetic algorithms for estimation problems with multiple optima, nondifferentiability, and other irregular features. *Journal of Business & Economic Statistics*, 13, 53-66.
- Kalliovirta L., Meitz M. and Saikkonen P. 2015. Gaussian Mixture Autoregressive model for univariate time series. *Journal of Time Series Analysis*, 36(2), 247-266.
- Meitz M., Preve D., Saikkonen P. 2023. A mixture autoregressive model based on Student's t-distribution. *Communications in Statistics Theory and Methods*, **52**(2), 499-515.
- Monahan J.F. 1984. A Note on Enforcing Stationarity in Autoregressive-Moving Average Models. *Biometrica* **71**, 403-404.
- Nash J. 1990. Compact Numerical Methods for Computers. Linear algebra and Function Minimization. *Adam Hilger*.
- Patnaik L.M. and Srinivas M. 1994. Adaptive Probabilities of Crossover and Mutation in Genetic Algorithms. *Transactions on Systems, Man and Cybernetics* **24**, 656-667.
- Smith R.E., Dike B.A., Stegmann S.A. 1995. Fitness inheritance in genetic algorithms. *Proceedings of the 1995 ACM Symposium on Applied Computing*, 345-350.
- Virolainen S. 2022. A mixture autoregressive model based on Gaussian and Student's t-distributions. Studies in Nonlinear Dynamics & Econometrics, **26**(4) 559-580.

# See Also

GSMAR, iterate\_more,, stmar\_to\_gstmar, add\_data, profile\_logliks, swap\_parametrization, get\_gradient, simulate.gsmar, predict.gsmar, diagnostic\_plot, quantile\_residual\_tests, cond\_moments, uncond\_moments, LR\_test, Wald\_test

#### **Examples**

```
## These are long running examples that use parallel computing.
## The below examples take approximately 90 seconds to run.
## Note that the number of estimation rounds (ncalls) is relatively small
## in the below examples to reduce the time required for running the examples.
## For reliable results, a large number of estimation rounds is recommended!
```

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```
# GMAR model
fit12 <- fitGSMAR(data=simudata, p=1, M=2, model="GMAR", ncalls=4, seeds=1:4)</pre>
summary(fit12)
plot(fit12)
profile_logliks(fit12)
diagnostic_plot(fit12)
# StMAR model (large estimate of the degrees of freedom)
fit42t <- fitGSMAR(data=M10Y1Y, p=4, M=2, model="StMAR", ncalls=2, seeds=c(1, 6))</pre>
summary(fit42t) # Overly large 2nd regime degrees of freedom estimate!
fit42gs <- stmar_to_gstmar(fit42t) # Switch to G-StMAR model</pre>
summary(fit42gs) # An appropriate G-StMVAR model with one G and one t regime
plot(fit42gs)
# Restricted StMAR model
fit42r <- fitGSMAR(M10Y1Y, p=4, M=2, model="StMAR", restricted=TRUE,
                   ncalls=2, seeds=1:2)
fit42r
# G-StMAR model with one GMAR type and one StMAR type regime
fit42gs <- fitGSMAR(M10Y1Y, p=4, M=c(1, 1), model="G-StMAR",
                    ncalls=1, seeds=4)
fit42gs
# The following three examples demonstrate how to apply linear constraints
# to the autoregressive (AR) parameters.
# Two-regime GMAR p=2 model with the second AR coefficient of
# of the second regime contrained to zero.
C22 <- list(diag(1, ncol=2, nrow=2), as.matrix(c(1, 0)))
fit22c <- fitGSMAR(M10Y1Y, p=2, M=2, constraints=C22, ncalls=1, seeds=6)
fit22c
# StMAR(3, 1) model with the second order AR coefficient constrained to zero.
C31 <- list(matrix(c(1, 0, 0, 0, 0, 1), ncol=2))
fit31tc <- fitGSMAR(M10Y1Y, p=3, M=1, model="StMAR", constraints=C31,</pre>
                    ncalls=1, seeds=1)
fit31tc
# Such StMAR(3, 2) model that the AR coefficients are restricted to be
# the same for both regimes and the second AR coefficients are
# constrained to zero.
fit32rc <- fitGSMAR(M10Y1Y, p=3, M=2, model="StMAR", restricted=TRUE,</pre>
                    constraints=matrix(c(1, 0, 0, 0, 0, 1), ncol=2),
                    ncalls=1, seeds=1)
fit32rc
```

GAfit 19

# **Description**

GAfit estimates specified GMAR, StMAR, or G-StMAR model using a genetic algorithm. The employed genetic algorithm is designed to find starting values for gradient based methods.

## Usage

```
GAfit(
  data,
  р,
 Μ,
 model = c("GMAR", "StMAR", "G-StMAR"),
 restricted = FALSE,
  constraints = NULL,
 parametrization = c("intercept", "mean"),
  conditional = TRUE,
  ngen = 200,
  popsize,
  smart_mu = min(100, ceiling(0.5 * ngen)),
 mu_scale,
  sigma_scale,
  initpop = NULL,
  regime_force_scale = 1,
  red_criteria = c(0.05, 0.01),
  to_return = c("alt_ind", "best_ind"),
 minval,
  seed = NULL,
)
```

#### **Arguments**

data	a numeric vector or class 'ts' object containing the data. NA values are not supported.
р	a positive integer specifying the autoregressive order of the model.
М	<b>For GMAR and StMAR models:</b> a positive integer specifying the number of mixture components.
	For G-StMAR models: a size $(2 \times 1)$ integer vector specifying the number of <i>GMAR type</i> components M1 in the first element and <i>StMAR type</i> components M2 in the second element. The total number of mixture components is M=M1+M2.
model	is "GMAR", "StMAR", or "G-StMAR" model considered? In the G-StMAR model, the first M1 components are <i>GMAR type</i> and the rest M2 components are <i>StMAR type</i> .
restricted	a logical argument stating whether the AR coefficients $\phi_{m,1},,\phi_{m,p}$ are restricted to be the same for all regimes.
constraints	specifies linear constraints imposed to each regime's autoregressive parameters separately.

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For non-restricted models: a list of size  $(p \times q_m)$  constraint matrices  $C_m$  of full column rank satisfying  $\phi_m = C_m \psi_m$  for all m = 1, ..., M, where  $\phi_m = (\phi_{m,1}, ..., \phi_{m,p})$  and  $\psi_m = (\psi_{m,1}, ..., \psi_{m,q_m})$ .

**For restricted models:** a size  $(p \times q)$  constraint matrix C of full column rank satisfying  $\phi = C\psi$ , where  $\phi = (\phi_1, ..., \phi_p)$  and  $\psi = \psi_1, ..., \psi_q$ .

The symbol  $\phi$  denotes an AR coefficient. Note that regardless of any constraints, the autoregressive order is always p for all regimes. Ignore or set to NULL if applying linear constraints is not desired.

parametrization

is the model parametrized with the "intercepts"  $\phi_{m,0}$  or "means"  $\mu_m = \phi_{m,0}/(1-\sum \phi_{i,m})$ ?

conditional a logical argument specifying whether the conditional or exact log-likelihood function should be used.

ngen a positive integer specifying the number of generations to be ran through in the genetic algorithm.

a positive even integer specifying the population size in the genetic algorithm. Default is 10\*d where d is the number of parameters.

a positive integer specifying the generation after which the random mutations in the genetic algorithm are "smart". This means that mutating individuals will mostly mutate fairly close (or partially close) to the best fitting individual so far.

a real valued vector of length two specifying the mean (the first element) and standard deviation (the second element) of the normal distribution from which the  $\mu_m$  mean-parameters are generated in random mutations in the genetic algorithm. Default is c(mean(data), sd(data)). Note that the genetic algorithm optimizes with mean-parametrization even when parametrization=="intercept", but input (in initpop) and output (return value) parameter vectors may be intercept-parametrized.

a positive real number specifying the standard deviation of the (zero mean, positive only by taking absolute value) normal distribution from which the component variance parameters are generated in the random mutations in the genetic algorithm. Default is var(stats::ar(data, order.max=10)\$resid, na.rm=TRUE).

a list of parameter vectors from which the initial population of the genetic algorithm will be generated from. The parameter vectors should be of form...

For non-restricted models: Size  $(M(p+3)+M-M1-1\times1)$  vector  $\theta=(v_1,...,v_M,$ 

$$\alpha_1, ..., \alpha_{M-1}, \nu$$
) where

- $v_m = (\phi_{m,0}, \phi_m, \sigma_m^2)$
- $\phi_m = (\phi_{m,1}, ..., \phi_{m,p}), m = 1, ..., M$
- $\nu = (\nu_{M1+1}, ..., \nu_M)$
- M1 is the number of GMAR type regimes.

In the GMAR model, M1=M and the parameter  $\nu$  dropped. In the StMAR model, M1=0.

If the model imposes **linear constraints** on the autoregressive parameters: Replace the vectors  $\phi_m$  with the vectors  $\psi_m$  that satisfy  $\phi_m = C_m \psi_m$  (see the argument constraints).

popsize

smart\_mu

mu\_scale

sigma\_scale

initpop

For restricted models: Size  $(3M+M-M1+p-1\times1)$  vector  $\theta=(\phi_{1,0},...,\phi_{M,0},\phi,\sigma_1^2,...,\sigma_M^2,\alpha_1,...,\alpha_{M-1},\nu)$ , where  $\phi=(\phi_1,...,\phi_p)$  contains the AR coefficients, which are common for all regimes.

If the model imposes **linear constraints** on the autoregressive parameters: Replace the vector  $\phi$  with the vector  $\psi$  that satisfies  $\phi{=}C\psi$  (see the argument constraints).

Symbol  $\phi$  denotes an AR coefficient,  $\sigma^2$  a variance,  $\alpha$  a mixing weight, and  $\nu$  a degrees of freedom parameter. If parametrization=="mean", just replace each intercept term  $\phi_{m,0}$  with the regimewise mean  $\mu_m = \phi_{m,0}/(1-\sum \phi_{i,m})$ . In the **G-StMAR** model, the first M1 components are *GMAR type* and the rest M2 components are *StMAR type*. Note that in the case **M=1**, the mixing weight parameters  $\alpha$  are dropped, and in the case of **StMAR** or **G-StMAR** model, the degrees of freedom parameters  $\nu$  have to be larger than 2. If not specified (or NULL as is default), the initial population will be drawn randomly.

#### regime\_force\_scale

a non-negative real number specifying how much should natural selection favor individuals with less regimes that have almost all mixing weights (practically) at zero (see red\_criteria), i.e., with less "redundant regimes". Set to zero for no favoring or large number for heavy favoring. Without any favoring the genetic algorithm gets more often stuck in an area of the parameter space where some regimes are wasted, but with too much favoring the best genes might never mix into the population and the algorithm might converge poorly. Default is 1 and it gives 2x larger surviving probability weights for individuals with no wasted regimes compared to individuals with one wasted regime. Number 2 would give 3x larger probabilities etc.

red\_criteria

a length 2 numeric vector specifying the criteria that is used to determine whether a regime is redundant or not. Any regime m which satisfies sum(mixing\_weights[,m] > red\_criteria[1]) < red\_criteria[2]\*n\_obs will be considered "redundant". One should be careful when adjusting this argument (set c(0, 0) to fully disable the 'redundant regime' features from the algorithm).

to\_return

should the genetic algorithm return the best fitting individual which has the least "redundant" regimes ("alt\_ind") or the individual which has the highest log-likelihood in general ("best\_ind") but might have more wasted regimes?

minval

a real number defining the minimum value of the log-likelihood function that will be considered. Values smaller than this will be treated as they were minval and the corresponding individuals will never survive. The default is -(10^(ceiling(log10(length(dat.

+ 1) - 1), and one should be very careful if adjusting this.

seed

a single value, interpreted as an integer, or NULL, that sets seed for the random number generator in the beginning of the function call. If calling GAfit from fitGSMAR, use the argument seeds instead of passing the argument seed.

... We currently use this to catch deprecated arguments.

#### **Details**

The core of the genetic algorithm is mostly based on the description by *Dorsey and Mayer* (1995). It utilizes a slightly modified version of the individually adaptive crossover and mutation rates described by *Patnaik and Srinivas* (1994) and employs (50%) fitness inheritance discussed by *Smith*,

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*Dike and Stegmann* (1995). Large (in absolute value) but stationary AR parameter values are generated with the algorithm proposed by Monahan (1984).

By "redundant" or "wasted" regimes we mean regimes that have the time varying mixing weights basically at zero for all t. The model with redundant regimes would have approximately the same log-likelihood value without the redundant regimes and there is no purpose to have redundant regimes in the model.

#### Value

Returns estimated parameter vector with the form described in initpop.

#### References

- Dorsey R. E. and Mayer W. J. 1995. Genetic algorithms for estimation problems with multiple optima, nondifferentiability, and other irregular features. *Journal of Business & Economic Statistics*, **13**, 53-66.
  - Kalliovirta L., Meitz M. and Saikkonen P. 2015. Gaussian Mixture Autoregressive model for univariate time series. *Journal of Time Series Analysis*, **36**(2), 247-266.
  - Meitz M., Preve D., Saikkonen P. 2023. A mixture autoregressive model based on Student's t-distribution. Communications in Statistics Theory and Methods, 52(2), 499-515.
  - Monahan J.F. 1984. A Note on Enforcing Stationarity in Autoregressive-Moving Average Models. *Biometrica* 71, 403-404.
  - Patnaik L.M. and Srinivas M. 1994. Adaptive Probabilities of Crossover and Mutation in Genetic Algorithms. *Transactions on Systems, Man and Cybernetics* 24, 656-667.
  - Smith R.E., Dike B.A., Stegmann S.A. 1995. Fitness inheritance in genetic algorithms. *Proceedings of the 1995 ACM Symposium on Applied Computing*, 345-350.
  - Virolainen S. 2022. A mixture autoregressive model based on Gaussian and Student's t-distributions. Studies in Nonlinear Dynamics & Econometrics, 26(4) 559-580.

## **Examples**

```
## These are long running examples

# Preliminary estimation of GMAR p=1, M=2, model with the genetic algorithm
# using only 100 generations (200 is recommended):
pars12_ga <- GAfit(data=simudata, p=1, M=2, model="GMAR", ngen=100, seed=1)
pars12_ga # Returns a parameter vector, not a class 'gsmar' object.</pre>
```

get\_ar\_roots

Calculate absolute values of the roots of the AR characteristic polynomials

#### Description

get\_ar\_roots calculates the absolute values of the roots of the AR characteristic polynomials for each mixture component.

get\_regime\_autocovs 23

## Usage

```
get_ar_roots(gsmar)
```

## **Arguments**

gsmar

a class 'gsmar' object, typically generated by fitGSMAR or GSMAR.

#### Value

Returns a list with M elements each containing the absolute values of the roots of the AR characteristic polynomial corresponding to each mixture component.

#### References

- Kalliovirta L., Meitz M. and Saikkonen P. 2015. Gaussian Mixture Autoregressive model for univariate time series. *Journal of Time Series Analysis*, **36**(2), 247-266.
- Meitz M., Preve D., Saikkonen P. 2023. A mixture autoregressive model based on Student's t-distribution. *Communications in Statistics Theory and Methods*, **52**(2), 499-515.
- Virolainen S. 2022. A mixture autoregressive model based on Gaussian and Student's t-distributions. Studies in Nonlinear Dynamics & Econometrics, 26(4) 559-580.

## **Examples**

```
params12 <- c(1.70, 0.85, 0.30, 4.12, 0.73, 1.98, 0.63)
gmar12 <- GSMAR(data=simudata, p=1, M=2, params=params12, model="GMAR")
get_ar_roots(gmar12)</pre>
```

get\_regime\_autocovs

Calculate regime specific autocovariances  $\gamma_m$ , p

## **Description**

get\_regime\_autocovs calculates the first p regime specific autocovariances  $\gamma_{m,p}$  for the given GMAR, StMAR, or G-StMAR model.

# Usage

```
get_regime_autocovs(gsmar)
```

#### **Arguments**

gsmar

a class 'gsmar' object, typically generated by fitGSMAR or GSMAR.

## Value

Returns a size (pxM) matrix containing the first p autocovariances of the components processes: i:th autocovariance in the i:th row and m:th component process in the m:th column.

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## References

- Kalliovirta L., Meitz M. and Saikkonen P. 2015. Gaussian Mixture Autoregressive model for univariate time series. *Journal of Time Series Analysis*, 36(2), 247-266.
- Meitz M., Preve D., Saikkonen P. 2023. A mixture autoregressive model based on Student's t-distribution. *Communications in Statistics Theory and Methods*, **52**(2), 499-515.
- Virolainen S. 2022. A mixture autoregressive model based on Gaussian and Student's t-distributions. Studies in Nonlinear Dynamics & Econometrics, 26(4) 559-580.
- Lütkepohl H. 2005. New Introduction to Multiple Time Series Analysis. Springer.

#### See Also

Other moment functions: cond\_moments(), get\_regime\_means(), get\_regime\_vars(), uncond\_moments()

## **Examples**

```
# GMAR model
params13 <- c(1.4, 0.88, 0.26, 2.46, 0.82, 0.74, 5.0, 0.68, 5.2, 0.72, 0.2)
gmar13 <- GSMAR(p=1, M=3, params=params13, model="GMAR")
get_regime_autocovs(gmar13)

# StMAR model
params12t <- c(1.38, 0.88, 0.27, 3.8, 0.74, 3.15, 0.8, 100, 3.6)
stmar12t <- GSMAR(p=1, M=2, params=params12t, model="StMAR")
get_regime_autocovs(stmar12t)

# G-StMAR model (similar to the StMAR model above)
params12gs <- c(1.38, 0.88, 0.27, 3.8, 0.74, 3.15, 0.8, 3.6)
gstmar12 <- GSMAR(p=1, M=c(1, 1), params=params12gs, model="G-StMAR")
get_regime_autocovs(gstmar12)
```

get\_regime\_means

Calculate regime specific means  $\mu_m$ 

## **Description**

```
get_regime_means calculates the regime means \mu_m=\phi_{m,0}/(1-\sum\phi_{i,m}) for the given GMAR, StMAR, or G-StMAR model
```

## Usage

```
get_regime_means(gsmar)
```

## **Arguments**

gsmar

a class 'gsmar' object, typically generated by fitGSMAR or GSMAR.

get\_regime\_vars 25

#### Value

Returns a length M vector containing the regime mean  $\mu_m$  in the m:th element.

#### References

- Kalliovirta L., Meitz M. and Saikkonen P. 2015. Gaussian Mixture Autoregressive model for univariate time series. *Journal of Time Series Analysis*, 36(2), 247-266.
- Meitz M., Preve D., Saikkonen P. 2023. A mixture autoregressive model based on Student's t-distribution. *Communications in Statistics Theory and Methods*, **52**(2), 499-515.
- Virolainen S. 2022. A mixture autoregressive model based on Gaussian and Student's t-distributions. Studies in Nonlinear Dynamics & Econometrics, 26(4) 559-580.

#### See Also

```
cond_moments, uncond_moments, get_regime_vars, get_regime_autocovs
Other moment functions: cond_moments(), get_regime_autocovs(), get_regime_vars(), uncond_moments()
```

## **Examples**

```
# GMAR model
params13 <- c(1.4, 0.88, 0.26, 2.46, 0.82, 0.74, 5.0, 0.68, 5.2, 0.72, 0.2)
gmar13 <- GSMAR(p=1, M=3, params=params13, model="GMAR")
get_regime_means(gmar13)

# StMAR model
params12t <- c(1.38, 0.88, 0.27, 3.8, 0.74, 3.15, 0.8, 100, 3.6)
stmar12t <- GSMAR(p=1, M=2, params=params12t, model="StMAR")
get_regime_means(stmar12t)

# G-StMAR model (similar to the StMAR model above)
params12gs <- c(1.38, 0.88, 0.27, 3.8, 0.74, 3.15, 0.8, 3.6)
gstmar12 <- GSMAR(p=1, M=c(1, 1), params=params12gs, model="G-StMAR")
get_regime_means(gstmar12)
```

get\_regime\_vars

Calculate regime specific variances  $\gamma_m$ , 0

## **Description**

get\_regime\_vars calculates the unconditional regime specific variances  $\gamma_{m,0}$  for the given GMAR, StMAR, or G-StMAR model.

# Usage

```
get_regime_vars(gsmar)
```

## **Arguments**

gsmar

a class 'gsmar' object, typically generated by fitGSMAR or GSMAR.

#### Value

Returns a length M vector containing the unconditional variances of the components processes: m:th element for the m:th regime.

#### References

- Kalliovirta L., Meitz M. and Saikkonen P. 2015. Gaussian Mixture Autoregressive model for univariate time series. *Journal of Time Series Analysis*, **36**(2), 247-266.
- Meitz M., Preve D., Saikkonen P. 2023. A mixture autoregressive model based on Student's t-distribution. *Communications in Statistics Theory and Methods*, **52**(2), 499-515.
- Virolainen S. 2022. A mixture autoregressive model based on Gaussian and Student's t-distributions. Studies in Nonlinear Dynamics & Econometrics, 26(4) 559-580.
- Lütkepohl H. 2005. New Introduction to Multiple Time Series Analysis. Springer.

#### See Also

Other moment functions: cond\_moments(), get\_regime\_autocovs(), get\_regime\_means(), uncond\_moments()

## **Examples**

```
# GMAR model
params13 <- c(1.4, 0.88, 0.26, 2.46, 0.82, 0.74, 5.0, 0.68, 5.2, 0.72, 0.2)
gmar13 <- GSMAR(p=1, M=3, params=params13, model="GMAR")
get_regime_vars(gmar13)

# StMAR model
params12t <- c(1.38, 0.88, 0.27, 3.8, 0.74, 3.15, 0.8, 100, 3.6)
stmar12t <- GSMAR(p=1, M=2, params=params12t, model="StMAR")
get_regime_vars(stmar12t)

# G-StMAR model (similar to the StMAR model above)
params12gs <- c(1.38, 0.88, 0.27, 3.8, 0.74, 3.15, 0.8, 3.6)
gstmar12 <- GSMAR(p=1, M=c(1, 1), params=params12gs, model="G-StMAR")
get_regime_vars(gstmar12)
```

**GSMAR** 

Create object of class 'gsmar' defining a GMAR, StMAR, or G-StMAR model

# Description

GSMAR creates an S3 object of class 'gsmar' that defines a GMAR, StMAR, or G-StMAR model.

## Usage

```
GSMAR(
  data,
  p,
 Μ,
 params,
 model = c("GMAR", "StMAR", "G-StMAR"),
  restricted = FALSE,
  constraints = NULL,
  conditional = TRUE,
  parametrization = c("intercept", "mean"),
  calc_qresiduals,
  calc_cond_moments,
  calc_std_errors = FALSE,
  custom_h = NULL
)
## S3 method for class 'gsmar'
logLik(object, ...)
## S3 method for class 'gsmar'
residuals(object, ...)
## S3 method for class 'gsmar'
summary(object, ..., digits = 2)
## S3 method for class 'gsmar'
plot(x, ..., include\_dens = TRUE)
## S3 method for class 'gsmar'
print(x, ..., digits = 2, summary_print = FALSE)
```

# Arguments

data a numeric vector or class 'ts' object containing the data. NA values are not

supported.

p a positive integer specifying the autoregressive order of the model.

M **For GMAR and StMAR models:** a positive integer specifying the number of mixture components.

For G-StMAR models: a size  $(2 \times 1)$  integer vector specifying the number of *GMAR type* components M1 in the first element and *StMAR type* components M2 in the second element. The total number of mixture components is M=M1+M2.

params a real valued parameter vector specifying the model.

For non-restricted models: Size  $(M(p+3)+M-M1-1\times1)$  vector  $\theta=(v_1,...,v_M, \alpha_1,...,\alpha_{M-1},\nu)$  where  $\bullet \ v_m=(\phi_{m,0},\phi_m,\sigma_m^2)$ 

- $\phi_m = (\phi_{m,1}, ..., \phi_{m,p}), m = 1, ..., M$
- $\nu = (\nu_{M1+1}, ..., \nu_M)$
- M1 is the number of GMAR type regimes.

In the GMAR model, M1 = M and the parameter  $\nu$  dropped. In the **StMAR** model, M1 = 0.

If the model imposes **linear constraints** on the autoregressive parameters: Replace the vectors  $\phi_m$  with the vectors  $\psi_m$  that satisfy  $\phi_m = C_m \psi_m$  (see the argument constraints).

For restricted models: Size  $(3M+M-M1+p-1\times1)$  vector  $\theta=(\phi_{1,0},...,\phi_{M,0},\phi,$  $\sigma_1^2,...,\sigma_M^2,\alpha_1,...,\alpha_{M-1},\nu$ ), where  $\phi=(\phi_1,...,\phi_p)$  contains the AR coefficients, which are common for all regimes.

If the model imposes **linear constraints** on the autoregressive parameters: Replace the vector  $\phi$  with the vector  $\psi$  that satisfies  $\phi = C\psi$  (see the argument constraints).

Symbol  $\phi$  denotes an AR coefficient,  $\sigma^2$  a variance,  $\alpha$  a mixing weight, and  $\nu$ a degrees of freedom parameter. If parametrization=="mean", just replace each intercept term  $\phi_{m,0}$  with the regimewise mean  $\mu_m = \phi_{m,0}/(1-\sum \phi_{i,m})$ . In the **G-StMAR** model, the first M1 components are *GMAR type* and the rest M2 components are StMAR type. Note that in the case M=1, the mixing weight parameters  $\alpha$  are dropped, and in the case of **StMAR** or **G-StMAR** model, the degrees of freedom parameters  $\nu$  have to be larger than 2.

mode1

is "GMAR", "StMAR", or "G-StMAR" model considered? In the G-StMAR model, the first M1 components are GMAR type and the rest M2 components are StMAR type.

restricted

a logical argument stating whether the AR coefficients  $\phi_{m,1},...,\phi_{m,p}$  are restricted to be the same for all regimes.

constraints

specifies linear constraints imposed to each regime's autoregressive parameters separately.

For non-restricted models: a list of size  $(p \times q_m)$  constraint matrices  $C_m$  of full column rank satisfying  $\phi_m = C_m \psi_m$  for all m = 1, ..., M, where  $\phi_m =$  $(\phi_{m,1},...,\phi_{m,p})$  and  $\psi_m = (\psi_{m,1},...,\psi_{m,q_m}).$ 

For restricted models: a size  $(p \times q)$  constraint matrix C of full column rank satisfying  $\phi = C\psi$ , where  $\phi = (\phi_1, ..., \phi_p)$  and  $\psi = \psi_1, ..., \psi_q$ .

The symbol  $\phi$  denotes an AR coefficient. Note that regardless of any constraints, the autoregressive order is always p for all regimes. Ignore or set to NULL if applying linear constraints is not desired.

conditional

a logical argument specifying whether the conditional or exact log-likelihood function should be used.

parametrization

is the model parametrized with the "intercepts"  $\phi_{m,0}$  or "means"  $\mu_m = \phi_{m,0}/(1 \sum \phi_{i,m}$ )?

calc\_gresiduals

should quantile residuals be calculated? Default is TRUE iff the model contains data.

calc\_cond\_moments

should conditional means and variances be calculated? Default is TRUE iff the model contains data.

calc\_std\_errors

should approximate standard errors be calculated?

custom\_h A numeric vector with same the length as the parameter vector: i:th element of

custom\_h is the difference used in central difference approximation for partial differentials of the log-likelihood function for the i:th parameter. If NULL (default), then the difference used for differentiating overly large degrees of freedom parameters is adjusted to avoid numerical problems, and the difference is

6e-6 for the other parameters.

object of class 'gsmar' created with fitGSMAR or GSMAR.

... in the plot method: arguments passed to the function density which calculates

the kernel density estimate of the data.

digits number of digits to be printed (max 20)

x object of class 'gsmar' created with fitGSMAR or GSMAR.

include\_dens Plot also kernel density estimate of the data and model implied stationary density

with regimewise densities? See the details.

summary\_print if set to TRUE then the print will include approximate standard errors for the

estimates, log-likelihood, information criteria values, modulus of the roots of the characteristic AR polynomials for each regime, and several unconditional

moments.

## **Details**

Models can be built without data, e.q., in order to simulate from the process, but some things such as quantile residuals and conditional moments can't be calculated without data.

If include\_dens == TRUE, the kernel density estimate of the data is calculated with the function density from the package stats. By default, the default settings of that function are used, including the usage of Gaussian kernel. Use the dot parameters to adjust the settings if desired.

By the model implied stationary density we mean the stationary one-dimensional mixture density of M regimes (see KMS 2015, Theorem 1 and the discussion following it for the Gaussian case and Theorem 2 in PMS 2018 for the Student's t case). The regimewise densities (i.e. each density 1,...,M in the stationary mixture density) are multiplied with the mixing weight parameters accordingly.

In the density plot black represents the kernel density estimate of the data, grey dashed line the model implied density, and the colored dotted lines the regime wise densities.

## Value

Returns an object of class 'gsmar' defining the specified GMAR, StMAR, or G-StMAR model. If data is supplied, the returned object contains (by default) empirical mixing weights, some conditional and unconditional moments, and quantile residuals. Note that the first p observations are taken as the initial values so the mixing weights, conditional moments, and quantile residuals start from the p+1:th observation (interpreted as t=1).

#### **Functions**

- logLik(gsmar): Log-likelihood method
- residuals(gsmar): residuals method to extract quantile residuals
- summary (gsmar): summary method, standard errors in brackets
- plot(gsmar): Plot method for class 'gsmar'
- print(gsmar): print method

#### References

- Kalliovirta L., Meitz M. and Saikkonen P. 2015. Gaussian Mixture Autoregressive model for univariate time series. *Journal of Time Series Analysis*, **36**(2), 247-266.
- Meitz M., Preve D., Saikkonen P. 2023. A mixture autoregressive model based on Student's t-distribution. *Communications in Statistics Theory and Methods*, **52**(2), 499-515.
- Virolainen S. 2022. A mixture autoregressive model based on Gaussian and Student's t-distributions. Studies in Nonlinear Dynamics & Econometrics, 26(4) 559-580.

#### See Also

```
fitGSMAR, iterate_more, add_data, stmar_to_gstmar, swap_parametrization, get_gradient,
simulate.gsmar, predict.gsmar, cond_moments, uncond_moments, LR_test, Wald_test
```

# **Examples**

```
# GMAR model without data
params22 <- c(0.9, 0.4, 0.2, 0.5, 0.7, 0.5, -0.2, 0.7, 0.7)
gmar22 <- GSMAR(p=2, M=2, params=params22, model="GMAR")</pre>
gmar22
# StMAR model, without data
params12t <- c(1.38, 0.88, 0.27, 3.8, 0.74, 3.15, 0.8, 300, 3.6)
stmar12t <- GSMAR(p=1, M=2, params=params12t, model="StMAR")</pre>
stmar12t
# G-StMAR model with data
params42gs <- c(0.04, 1.34, -0.59, 0.54, -0.36, 0.01, 0.06, 1.28, -0.36,
                0.2, -0.15, 0.04, 0.19, 9.75)
gstmar42 <- GSMAR(data=M10Y1Y, p=4, M=c(1, 1), params=params42gs,</pre>
                  model="G-StMAR")
gstmar42
# Restricted G-StMAR model with data
params42gsr <- c(0.13, 0.03, 1.29, -0.4, 0.25, -0.2, 0.03, 0.05, 0.51, 2.76)
gstmar42r <- GSMAR(data=M10Y1Y, p=4, M=c(1, 1), params=params42gsr,</pre>
                   model="G-StMAR", restricted=TRUE)
gstmar42r
```

is\_stationary 31

is\_stationary Check the stationary condition of specified GMAR, StMAR, or G-StMAR model.

## **Description**

is\_stationary checks the stationarity condition of the specified GMAR, StMAR, or G-StMAR model.

## Usage

```
is_stationary(
  p,
  M,
  params,
  model = c("GMAR", "StMAR", "G-StMAR"),
  restricted = FALSE,
  constraints = NULL
)
```

## Arguments

р

a positive integer specifying the autoregressive order of the model.

М

**For GMAR and StMAR models:** a positive integer specifying the number of mixture components.

For G-StMAR models: a size  $(2 \times 1)$  integer vector specifying the number of *GMAR type* components M1 in the first element and *StMAR type* components M2 in the second element. The total number of mixture components is M=M1+M2.

params

a real valued parameter vector specifying the model.

For non-restricted models: Size  $(M(p+3)+M-M1-1\times 1)$  vector  $\theta=(v_1,...,v_M,$ 

 $\alpha_1,...,\alpha_{M-1},\nu)$  where

- $v_m = (\phi_{m,0}, \phi_m, \sigma_m^2)$
- $\phi_m = (\phi_{m,1}, ..., \phi_{m,p}), m = 1, ..., M$
- $\nu = (\nu_{M1+1}, ..., \nu_M)$
- M1 is the number of GMAR type regimes.

In the GMAR model, M1=M and the parameter  $\nu$  dropped. In the StMAR model, M1=0.

If the model imposes **linear constraints** on the autoregressive parameters: Replace the vectors  $\phi_m$  with the vectors  $\psi_m$  that satisfy  $\phi_m = C_m \psi_m$  (see the argument constraints).

For restricted models: Size  $(3M+M-M1+p-1\times1)$  vector  $\theta=(\phi_{1,0},...,\phi_{M,0},\phi,\sigma_1^2,...,\sigma_M^2,\alpha_1,...,\alpha_{M-1},\nu)$ , where  $\phi=(\phi_1,...,\phi_p)$  contains the AR coefficients, which are common for all regimes.

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> If the model imposes **linear constraints** on the autoregressive parameters: Replace the vector  $\phi$  with the vector  $\psi$  that satisfies  $\phi = C\psi$  (see the argument constraints).

Symbol  $\phi$  denotes an AR coefficient,  $\sigma^2$  a variance,  $\alpha$  a mixing weight, and  $\nu$ a degrees of freedom parameter. If parametrization=="mean", just replace each intercept term  $\phi_{m,0}$  with the regimewise mean  $\mu_m = \phi_{m,0}/(1-\sum \phi_{i,m})$ . In the **G-StMAR** model, the first M1 components are *GMAR type* and the rest M2 components are StMAR type. Note that in the case M=1, the mixing weight parameters  $\alpha$  are dropped, and in the case of **StMAR** or **G-StMAR** model, the degrees of freedom parameters  $\nu$  have to be larger than 2.

model

is "GMAR", "StMAR", or "G-StMAR" model considered? In the G-StMAR model, the first M1 components are GMAR type and the rest M2 components are StMAR type.

a logical argument stating whether the AR coefficients  $\phi_{m,1},...,\phi_{m,p}$  are restricted to be the same for all regimes.

constraints

specifies linear constraints imposed to each regime's autoregressive parameters separately.

For non-restricted models: a list of size  $(p \times q_m)$  constraint matrices  $C_m$  of full column rank satisfying  $\phi_m = C_m \psi_m$  for all m = 1, ..., M, where  $\phi_m =$  $(\phi_{m,1},...,\phi_{m,p})$  and  $\psi_m = (\psi_{m,1},...,\psi_{m,q_m}).$ 

For restricted models: a size  $(p \times q)$  constraint matrix C of full column rank satisfying  $\phi = C\psi$ , where  $\phi = (\phi_1, ..., \phi_p)$  and  $\psi = \psi_1, ..., \psi_q$ .

The symbol  $\phi$  denotes an AR coefficient. Note that regardless of any constraints, the autoregressive order is always p for all regimes. Ignore or set to NULL if applying linear constraints is not desired.

#### **Details**

This function falsely returns FALSE for stationary models when the parameter is extremely close to the boundary of the stationarity region.

#### Value

Returns TRUE or FALSE accordingly.

## References

- Kalliovirta L., Meitz M. and Saikkonen P. 2015. Gaussian Mixture Autoregressive model for univariate time series. Journal of Time Series Analysis, **36**(2), 247-266.
- Meitz M., Preve D., Saikkonen P. 2023. A mixture autoregressive model based on Student's t-distribution. Communications in Statistics - Theory and Methods, **52**(2), 499-515.
- Virolainen S. 2022. A mixture autoregressive model based on Gaussian and Student's tdistributions. Studies in Nonlinear Dynamics & Econometrics, 26(4) 559-580.

restricted

iterate\_more 33

## **Examples**

```
# GMAR model
params22 \leftarrow c(0.4, 0.39, 0.6, 0.3, 0.4, 0.1, 0.6, 0.3, 0.8)
is_stationary(p=2, M=2, params=params22)
# StMAR model
params12t <- c(-0.3, 1, 0.9, 0.1, 0.8, 0.6, 0.7, 10, 12)
is_stationary(p=1, M=2, params=params12t, model="StMAR")
# G-StMAR model
params12gs \leftarrow c(1, 0.1, 1, 2, 0.2, 2, 0.8, 20)
is_stationary(p=1, M=c(1, 1), params=params12gs, model="G-StMAR")
# Restricted GMAR model
params13r < c(0.1, 0.2, 0.3, -0.99, 0.1, 0.2, 0.3, 0.5, 0.3)
is_stationary(p=1, M=3, params=params13r, restricted=TRUE)
# Such StMAR(3, 2) that the AR coefficients are restricted to be the
# same for both regimes and that the second AR coefficients are
# constrained to zero.
params32trc <- c(1, 2, 0.8, -0.3, 1, 2, 0.7, 11, 12)
is_stationary(p=3, M=2, params=params32trc, model="StMAR", restricted=TRUE,
              constraints=matrix(c(1, 0, 0, 0, 0, 1), ncol=2))
```

iterate\_more

Maximum likelihood estimation of GMAR, StMAR, or G-StMAR model with preliminary estimates

## Description

iterate\_more uses a variable metric algorithm to finalize maximum likelihood estimation of a GMAR, StMAR or G-StMAR model (object of class 'gsmar') which already has preliminary estimates.

## Usage

```
iterate_more(gsmar, maxit = 100, custom_h = NULL, calc_std_errors = TRUE)
```

# Arguments

gsmar a class 'gsmar' object, typically generated by fitGSMAR or GSMAR.

maxit the maximum number of iterations for the variable metric algorithm.

custom\_h A numeric vector with same the length as the parameter vector: i:th element of

custom\_h is the difference used in central difference approximation for partial differentials of the log-likelihood function for the i:th parameter. If NULL (default), then the difference used for differentiating overly large degrees of freedom parameters is adjusted to avoid numerical problems, and the difference is 6e-6 for the other parameters.

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```
calc_std_errors
```

should approximate standard errors be calculated?

#### **Details**

The main purpose of iterate\_more is to provide a simple and convenient tool to finalize the estimation when the maximum number of iterations is reached when estimating a model with the main estimation function fitGSMAR. iterate\_more is essentially a wrapper for the functions optim from the package stats and GSMAR from the package uGMAR.

#### Value

Returns an object of class 'gsmar' defining the estimated model.

#### References

- Kalliovirta L., Meitz M. and Saikkonen P. 2015. Gaussian Mixture Autoregressive model for univariate time series. *Journal of Time Series Analysis*, **36**(2), 247-266.
- Meitz M., Preve D., Saikkonen P. 2023. A mixture autoregressive model based on Student's t-distribution. *Communications in Statistics Theory and Methods*, **52**(2), 499-515.
- Virolainen S. 2022. A mixture autoregressive model based on Gaussian and Student's tdistributions. Studies in Nonlinear Dynamics & Econometrics, 26(4) 559-580.

#### See Also

```
fitGSMAR, GSMAR, stmar_to_gstmar, profile_logliks, optim
```

## **Examples**

```
# Estimate GMAR model with on only 1 iteration in variable metric algorithm
fit12 <- fitGSMAR(simudata, p=1, M=2, maxit=1, ncalls=1, seeds=1)
fit12

# Iterate more since iteration limit was reached
fit12 <- iterate_more(fit12)
fit12</pre>
```

loglikelihood

Compute the log-likelihood of GMAR, StMAR, or G-StMAR model

## **Description**

loglikelihood computes the log-likelihood of the specified GMAR, StMAR, or G-StMAR model. Exists for convenience if one wants to for example plot profile log-likelihoods or employ other estimation algorithms. Use minval to control what happens when the parameter vector is outside the parameter space.

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## Usage

```
loglikelihood(
  data,
  p,
  M,
  params,
  model = c("GMAR", "StMAR", "G-StMAR"),
  restricted = FALSE,
  constraints = NULL,
  conditional = TRUE,
  parametrization = c("intercept", "mean"),
  return_terms = FALSE,
  minval = NA
)
```

## **Arguments**

data

a numeric vector or class 'ts' object containing the data. NA values are not supported.

р

a positive integer specifying the autoregressive order of the model.

Μ

**For GMAR and StMAR models:** a positive integer specifying the number of mixture components.

For G-StMAR models: a size  $(2 \times 1)$  integer vector specifying the number of *GMAR type* components M1 in the first element and *StMAR type* components M2 in the second element. The total number of mixture components is M=M1+M2.

params

a real valued parameter vector specifying the model.

For non-restricted models: Size  $(M(p+3)+M-M1-1\times1)$  vector  $\theta=(v_1,...,v_M,$ 

 $\alpha_1, ..., \alpha_{M-1}, \nu$ ) where

- $v_m = (\phi_{m,0}, \phi_m, \sigma_m^2)$
- $\phi_m = (\phi_{m,1}, ..., \phi_{m,p}), m = 1, ..., M$
- $\nu = (\nu_{M1+1}, ..., \nu_M)$
- M1 is the number of GMAR type regimes.

In the GMAR model, M1=M and the parameter  $\nu$  dropped. In the StMAR model, M1=0.

If the model imposes **linear constraints** on the autoregressive parameters: Replace the vectors  $\phi_m$  with the vectors  $\psi_m$  that satisfy  $\phi_m = C_m \psi_m$  (see the argument constraints).

For restricted models: Size  $(3M+M-M1+p-1\times1)$  vector  $\theta=(\phi_{1,0},...,\phi_{M,0},\phi,\sigma_1^2,...,\sigma_M^2,\alpha_1,...,\alpha_{M-1},\nu)$ , where  $\phi=(\phi_1,...,\phi_p)$  contains the AR coefficients, which are common for all regimes.

If the model imposes **linear constraints** on the autoregressive parameters: Replace the vector  $\phi$  with the vector  $\psi$  that satisfies  $\phi = C\psi$  (see the argument constraints).

Symbol  $\phi$  denotes an AR coefficient,  $\sigma^2$  a variance,  $\alpha$  a mixing weight, and  $\nu$  a degrees of freedom parameter. If parametrization=="mean", just replace

. . . . . . . . .

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each intercept term  $\phi_{m,0}$  with the regimewise mean  $\mu_m = \phi_{m,0}/(1-\sum\phi_{i,m})$ . In the **G-StMAR** model, the first M1 components are *GMAR type* and the rest M2 components are *StMAR type*. Note that in the case **M=1**, the mixing weight parameters  $\alpha$  are dropped, and in the case of **StMAR** or **G-StMAR** model, the degrees of freedom parameters  $\nu$  have to be larger than 2.

model

is "GMAR", "StMAR", or "G-StMAR" model considered? In the G-StMAR model, the first M1 components are *GMAR type* and the rest M2 components are *StMAR type*.

restricted

a logical argument stating whether the AR coefficients  $\phi_{m,1},...,\phi_{m,p}$  are restricted to be the same for all regimes.

constraints

specifies linear constraints imposed to each regime's autoregressive parameters separately.

For non-restricted models: a list of size  $(p \times q_m)$  constraint matrices  $C_m$  of full column rank satisfying  $\phi_m = C_m \psi_m$  for all m = 1, ..., M, where  $\phi_m = (\phi_{m,1}, ..., \phi_{m,p})$  and  $\psi_m = (\psi_{m,1}, ..., \psi_{m,q_m})$ .

For restricted models: a size  $(p \times q)$  constraint matrix C of full column rank satisfying  $\phi = C\psi$ , where  $\phi = (\phi_1, ..., \phi_p)$  and  $\psi = \psi_1, ..., \psi_q$ .

The symbol  $\phi$  denotes an AR coefficient. Note that regardless of any constraints, the autoregressive order is always p for all regimes. Ignore or set to NULL if applying linear constraints is not desired.

conditional

a logical argument specifying whether the conditional or exact log-likelihood function should be used.

parametrization

is the model parametrized with the "intercepts"  $\phi_{m,0}$  or "means"  $\mu_m = \phi_{m,0}/(1-\sum \phi_{i,m})$ ?

return\_terms

should the terms  $l_t: t=1,..,T$  in the log-likelihood function (see KMS 2015, eq.(13) or MPS 2018, eq.(15)) be returned instead of the log-likelihood value?

minval

this will be returned when the parameter vector is outside the parameter space and boundaries==TRUE.

#### Value

Returns the log-likelihood value or the terms described in return\_terms.

#### References

- Galbraith, R., Galbraith, J. 1974. On the inverses of some patterned matrices arising in the theory of stationary time series. *Journal of Applied Probability* **11**, 63-71.
- Kalliovirta L. (2012) Misspecification tests based on quantile residuals. *The Econometrics Journal*, **15**, 358-393.
- Kalliovirta L., Meitz M. and Saikkonen P. 2015. Gaussian Mixture Autoregressive model for univariate time series. *Journal of Time Series Analysis*, **36**(2), 247-266.
- Meitz M., Preve D., Saikkonen P. 2023. A mixture autoregressive model based on Student's t-distribution. Communications in Statistics - Theory and Methods, 52(2), 499-515.
- Virolainen S. 2022. A mixture autoregressive model based on Gaussian and Student's t-distributions. Studies in Nonlinear Dynamics & Econometrics, 26(4) 559-580.

LR\_test 37

# See Also

fitGSMAR, GSMAR, quantile\_residuals, mixing\_weights, calc\_gradient

## **Examples**

```
# GMAR model
params12 <- c(1.70, 0.85, 0.30, 4.12, 0.73, 1.98, 0.63)
loglikelihood(simudata, p=1, M=2, params=params12, model="GMAR")

# G-StMAR-model
params42gs <- c(0.04, 1.34, -0.59, 0.54, -0.36, 0.01, 0.06, 1.28, -0.36, 0.2, -0.15, 0.04, 0.19, 9.75)
loglikelihood(M10Y1Y, p=4, M=c(1, 1), params=params42gs, model="G-StMAR")
```

LR\_test

Perform likelihood ratio test

# Description

LR\_test performs a likelihood ratio test for a GMAR, StMAR, or G-StMAR model.

# Usage

```
LR_test(gsmar1, gsmar2)
```

# **Arguments**

gsmar1	an object of class 'gsmar' generated by fitGSMAR or GSMAR, containing the
	unconstrained model.
gsmar2	an object of class 'gsmar' generated by fitGSMAR or GSMAR, containing the
	constrained model.

## **Details**

Performs a likelihood ratio test, testing the null hypothesis that the true parameter value lies in the constrained parameter space specified by constraints imposed to the model gsmar2. Under the null, the test statistic is asymptotically  $\chi^2$ -distributed with k degrees of freedom, k being the difference in the dimensions of the unconstrained and constrained parameter spaces.

Note that this function does **not** verify that the two models are actually nested. Notably, GSMAR models with different autoregressive orders are not nested, whereas testing models with different numbers of regimes induce an identification problem and thereby unreliable test results (see the discussion related to Theorem 2 in Virolainen, 2021).

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## Value

A list with class "htest" containing the following components:

statistic the value of the likelihood ratio statistics.

parameter the degrees of freedom of the likelihood ratio statistic.

p. value the p-value of the test.

alternative a character string describing the alternative hypothesis.

method a character string indicating the type of the test (likelihood ratio test).

data.name a character string giving the names of the supplied models, gsmar1 and gsmar2.

gsmar1 the supplied argument gsmar1
gsmar2 the supplied argument gsmar2

## References

- Kalliovirta L., Meitz M. and Saikkonen P. 2015. Gaussian Mixture Autoregressive model for univariate time series. *Journal of Time Series Analysis*, **36**(2), 247-266.
- Meitz M., Preve D., Saikkonen P. 2023. A mixture autoregressive model based on Student's t-distribution. *Communications in Statistics Theory and Methods*, **52**(2), 499-515.
- Virolainen S. 2022. A mixture autoregressive model based on Gaussian and Student's t-distributions. Studies in Nonlinear Dynamics & Econometrics, **26**(4) 559-580.

## See Also

```
Wald_test, fitGSMAR, GSMAR, diagnostic_plot, profile_logliks, quantile_residual_tests,
cond_moment_plot
```

# **Examples**

*M10Y1Y* 39

M10Y1Y

Spread between 10-Year and 1-Year Treasury rates: M10Y1Y

# **Description**

A dataset containing monthly U.S. interest rate spread between the 10-Year Treasury constant maturity rate and 1-Year Treasury constant maturity rate from 1982 January to 2020 December.

# Usage

M10Y1Y

#### **Format**

A class 'ts' time series object containing 468 observations.

## **Source**

https://fred.stlouisfed.org/series/GS10 https://fred.stlouisfed.org/series/GS1

mixing\_weights

Calculate mixing weights of GMAR, StMAR or G-StMAR model

# **Description**

mixing\_weights calculates the mixing weights of the specified GMAR, StMAR or G-StMAR model and returns them as a matrix.

# Usage

```
mixing_weights(
  data,
  p,
  M,
  params,
  model = c("GMAR", "StMAR", "G-StMAR"),
  restricted = FALSE,
  constraints = NULL,
  parametrization = c("intercept", "mean")
)
```

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## **Arguments**

data

a numeric vector or class 'ts' object containing the data. NA values are not supported.

р

a positive integer specifying the autoregressive order of the model.

М

**For GMAR and StMAR models:** a positive integer specifying the number of mixture components.

For G-StMAR models: a size  $(2 \times 1)$  integer vector specifying the number of *GMAR type* components M1 in the first element and *StMAR type* components M2 in the second element. The total number of mixture components is M=M1+M2.

params

a real valued parameter vector specifying the model.

For non-restricted models: Size  $(M(p+3)+M-M1-1\times 1)$  vector  $\theta=(v_1,...,v_M,$ 

 $\alpha_1,...,\alpha_{M-1},\nu)$  where

- $v_m = (\phi_{m,0}, \phi_m, \sigma_m^2)$
- $\phi_m = (\phi_{m,1}, ..., \phi_{m,p}), m = 1, ..., M$
- $\nu = (\nu_{M1+1}, ..., \nu_M)$
- M1 is the number of GMAR type regimes.

In the GMAR model, M1=M and the parameter  $\nu$  dropped. In the StMAR model, M1=0.

If the model imposes **linear constraints** on the autoregressive parameters: Replace the vectors  $\phi_m$  with the vectors  $\psi_m$  that satisfy  $\phi_m = C_m \psi_m$  (see the argument constraints).

For restricted models: Size  $(3M+M-M1+p-1\times1)$  vector  $\theta=(\phi_{1,0},...,\phi_{M,0},\phi,\sigma_1^2,...,\sigma_M^2,\alpha_1,...,\alpha_{M-1},\nu)$ , where  $\phi=(\phi_1,...,\phi_p)$  contains the AR coefficients, which are common for all regimes.

If the model imposes **linear constraints** on the autoregressive parameters: Replace the vector  $\phi$  with the vector  $\psi$  that satisfies  $\phi{=}C\psi$  (see the argument constraints).

Symbol  $\phi$  denotes an AR coefficient,  $\sigma^2$  a variance,  $\alpha$  a mixing weight, and  $\nu$  a degrees of freedom parameter. If parametrization=="mean", just replace each intercept term  $\phi_{m,0}$  with the regimewise mean  $\mu_m = \phi_{m,0}/(1-\sum\phi_{i,m})$ . In the **G-StMAR** model, the first M1 components are *GMAR type* and the rest M2 components are *StMAR type*. Note that in the case **M=1**, the mixing weight parameters  $\alpha$  are dropped, and in the case of **StMAR** or **G-StMAR** model, the degrees of freedom parameters  $\nu$  have to be larger than 2.

mode1

is "GMAR", "StMAR", or "G-StMAR" model considered? In the G-StMAR model, the first M1 components are *GMAR type* and the rest M2 components are *StMAR type*.

restricted

a logical argument stating whether the AR coefficients  $\phi_{m,1},...,\phi_{m,p}$  are restricted to be the same for all regimes.

constraints

specifies linear constraints imposed to each regime's autoregressive parameters separately.

For non-restricted models: a list of size  $(p \times q_m)$  constraint matrices  $C_m$  of full column rank satisfying  $\phi_m = C_m \psi_m$  for all m = 1, ..., M, where  $\phi_m = (\phi_{m,1}, ..., \phi_{m,p})$  and  $\psi_m = (\psi_{m,1}, ..., \psi_{m,q_m})$ .

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For restricted models: a size  $(p \times q)$  constraint matrix C of full column rank satisfying  $\phi = C\psi$ , where  $\phi = (\phi_1, ..., \phi_p)$  and  $\psi = \psi_1, ..., \psi_q$ .

The symbol  $\phi$  denotes an AR coefficient. Note that regardless of any constraints, the autoregressive order is always p for all regimes. Ignore or set to NULL if applying linear constraints is not desired.

parametrization

is the model parametrized with the "intercepts"  $\phi_{m,0}$  or "means"  $\mu_m = \phi_{m,0}/(1-\sum \phi_{i,m})$ ?

#### **Details**

The first p observations are taken to be the initial values.

## Value

If to\_return=="mw": a size  $((n_obs - p) \times M)$  matrix containing the mixing weights: for m:th component in m:th column.

If to\_return=="mw\_tplus1": a size  $((n_obs-p+1)\times M)$  matrix containing the mixing weights: for m:th component in m:th column. The last row is for  $\alpha_{m,T+1}$ .

#### References

- Galbraith, R., Galbraith, J. 1974. On the inverses of some patterned matrices arising in the theory of stationary time series. *Journal of Applied Probability* **11**, 63-71.
- Kalliovirta L. (2012) Misspecification tests based on quantile residuals. *The Econometrics Journal*, **15**, 358-393.
- Kalliovirta L., Meitz M. and Saikkonen P. 2015. Gaussian Mixture Autoregressive model for univariate time series. *Journal of Time Series Analysis*, **36**(2), 247-266.
- Meitz M., Preve D., Saikkonen P. 2023. A mixture autoregressive model based on Student's t-distribution. *Communications in Statistics Theory and Methods*, **52**(2), 499-515.
- Virolainen S. 2022. A mixture autoregressive model based on Gaussian and Student's t-distributions. Studies in Nonlinear Dynamics & Econometrics, **26**(4) 559-580.

# **Examples**

```
# GMAR model
params12 <- c(1.70, 0.85, 0.30, 4.12, 0.73, 1.98, 0.63)
mixing_weights(simudata, p=1, M=2, params=params12, model="GMAR")

# G-StMAR-model
params42gs <- c(0.04, 1.34, -0.59, 0.54, -0.36, 0.01, 0.06, 1.28, -0.36, 0.2, -0.15, 0.04, 0.19, 9.75)
mixing_weights(M10Y1Y, p=4, M=c(1, 1), params=params42gs, model="G-StMAR")
```

42 pick\_pars

pick\_pars

Pick  $\phi_0$  (or  $\mu$ ), AR-coefficients, and variance parameters from a parameter vector

# **Description**

pick\_pars picks  $\phi_0/\mu$ , AR-coefficients, and variance parameters from the given parameter vector.

## Usage

```
pick_pars(
   p,
   M,
   params,
   model = c("GMAR", "StMAR", "G-StMAR"),
   restricted = FALSE,
   constraints = NULL
)
```

# **Arguments**

р

a positive integer specifying the autoregressive order of the model.

М

**For GMAR and StMAR models:** a positive integer specifying the number of mixture components.

For G-StMAR models: a size  $(2 \times 1)$  integer vector specifying the number of *GMAR type* components M1 in the first element and *StMAR type* components M2 in the second element. The total number of mixture components is M=M1+M2.

params

a real valued parameter vector specifying the model.

For non-restricted models: Size  $(M(p+3)+M-M1-1\times1)$  vector  $\theta=(v_1,...,v_M,$ 

 $\alpha_1,...,\alpha_{M-1},\nu)$  where

- $v_m = (\phi_{m,0}, \phi_m, \sigma_m^2)$
- $\phi_m = (\phi_{m,1}, ..., \phi_{m,p}), m = 1, ..., M$
- $\nu = (\nu_{M1+1}, ..., \nu_M)$
- M1 is the number of GMAR type regimes.

In the GMAR model, M1=M and the parameter  $\nu$  dropped. In the StMAR model, M1=0.

If the model imposes **linear constraints** on the autoregressive parameters: Replace the vectors  $\phi_m$  with the vectors  $\psi_m$  that satisfy  $\phi_m = C_m \psi_m$  (see the argument constraints).

For restricted models: Size  $(3M+M-M1+p-1\times1)$  vector  $\theta=(\phi_{1,0},...,\phi_{M,0},\phi,\sigma_1^2,...,\sigma_M^2,\alpha_1,...,\alpha_{M-1},\nu)$ , where  $\phi=(\phi_1,...,\phi_p)$  contains the AR coefficients, which are common for all regimes.

If the model imposes **linear constraints** on the autoregressive parameters: Replace the vector  $\phi$  with the vector  $\psi$  that satisfies  $\phi$ = $C\psi$  (see the argument constraints).

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Symbol  $\phi$  denotes an AR coefficient,  $\sigma^2$  a variance,  $\alpha$  a mixing weight, and  $\nu$  a degrees of freedom parameter. If parametrization="mean", just replace each intercept term  $\phi_{m,0}$  with the regimewise mean  $\mu_m = \phi_{m,0}/(1-\sum\phi_{i,m})$ . In the **G-StMAR** model, the first M1 components are *GMAR type* and the rest M2 components are *StMAR type*. Note that in the case **M=1**, the mixing weight parameters  $\alpha$  are dropped, and in the case of **StMAR** or **G-StMAR** model, the degrees of freedom parameters  $\nu$  have to be larger than 2.

model

is "GMAR", "StMAR", or "G-StMAR" model considered? In the G-StMAR model, the first M1 components are *GMAR type* and the rest M2 components are *StMAR type*.

restricted

a logical argument stating whether the AR coefficients  $\phi_{m,1},...,\phi_{m,p}$  are restricted to be the same for all regimes.

constraints

specifies linear constraints imposed to each regime's autoregressive parameters separately.

For non-restricted models: a list of size  $(p \times q_m)$  constraint matrices  $C_m$  of full column rank satisfying  $\phi_m = C_m \psi_m$  for all m = 1, ..., M, where  $\phi_m = (\phi_{m,1}, ..., \phi_{m,p})$  and  $\psi_m = (\psi_{m,1}, ..., \psi_{m,q_m})$ .

For restricted models: a size  $(p \times q)$  constraint matrix C of full column rank satisfying  $\phi = C\psi$ , where  $\phi = (\phi_1, ..., \phi_p)$  and  $\psi = \psi_1, ..., \psi_q$ .

The symbol  $\phi$  denotes an AR coefficient. Note that regardless of any constraints, the autoregressive order is always p for all regimes. Ignore or set to NULL if applying linear constraints is not desired.

#### Value

Returns a  $((p+2) \times M)$  matrix containing the parameters, column for each component. The first row for  $\phi_0$  or  $\mu$  depending on the parametrization, the second row for  $\phi_1$ , ..., the second to last row for  $\phi_p$ , and the last row for  $\sigma^2$ . @keywords internal

plot.gsmarpred

Plot method for class 'gsmarpred' objects

# **Description**

plot.gsmarpred is plot method for class 'gsmarpred' objects

#### **Usage**

```
## S3 method for class 'gsmarpred'
plot(x, ..., nt, mix_weights = TRUE, add_grid = TRUE)
```

plot.qrtest

# **Arguments**

Χ	object of class 'gsmarpred' created with predict.gsmar.
	arguments passed to function grid.
nt	a positive integer specifying the number of observations to be plotted along with the prediction. Default is round(length(data)*0.15).
mix_weights	TRUE if forecasts for mixing weights should be plotted, FALSE in not.
add_grid	should grid a be added to the plots?

#### **Details**

This method is intended for plotting forecasts of GSMAR processes.

plot.qrtest Quantile residual tests for GMAR, StMAR, and G-StMAR models

# **Description**

quantile\_residual\_tests performs quantile residual tests for GMAR, StMAR, and G-StMAR models, testing normality, autocorrelation, and conditional heteroscedasticity of the quantile residuals.

# Usage

```
## S3 method for class 'qrtest'
plot(x, ...)

## S3 method for class 'qrtest'
print(x, ..., digits = 3)

quantile_residual_tests(
   gsmar,
   lags_ac = c(1, 3, 6, 12),
   lags_ch = lags_ac,
   nsimu = 1,
   print_res = TRUE
)
```

# Arguments

```
x object of class 'qrtest' created with the function quantile_residual_tests.
... graphical parameters passed to segments in plot.qrtest. Currently not used in print.qrtest
digits the number of digits to be print
gsmar a class 'gsmar' object, typically generated by fitGSMAR or GSMAR.
```

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lags_ac	a numeric vector of positive integers specifying the lags for which autocorrelation is tested.
lags_ch	a numeric vector of positive integers specifying the lags for which conditional heteroscedasticity is tested.
nsimu	a positive integer specifying to how many simulated observations the covariance matrix Omega (see Kalliovirta (2012)) should be based on. If smaller than data size, then omega will be based on the given data and not on simulated data. Having the covariance matrix omega based on a large simulated sample might improve the tests size properties.
print_res	a logical argument defining whether the results should be printed or not.

#### **Details**

For a correctly specified GSMAR model employing the maximum likelihood estimator, the quantile residuals are asymptotically independent with standard normal distribution. They can hence be used in a similar manner to conventional Pearson's residuals. For more details about quantile residual based diagnostics, and in particular, about the quantile residual tests, see the cited article by *Kalliovirta* (2012).

#### Value

Returns an object of class 'qrtest' containing the test results in data frames. In the cases of auto-correlation and conditional heteroscedasticity tests, the returned object also contains the associated individual statistics and their standard errors, discussed in *Kalliovirta* (2012) at the pages 369-370.

## **Functions**

- plot(qrtest): Plot p-values of the autocorrelation and conditional heteroskedasticity tests.
- print(qrtest): Print method for class 'qrtest' objects

## References

- Galbraith, R., Galbraith, J. 1974. On the inverses of some patterned matrices arising in the theory of stationary time series. *Journal of Applied Probability* **11**, 63-71.
- Kalliovirta L. (2012) Misspecification tests based on quantile residuals. The Econometrics Journal, 15, 358-393.
- Kalliovirta L., Meitz M. and Saikkonen P. 2015. Gaussian Mixture Autoregressive model for univariate time series. *Journal of Time Series Analysis*, **36**(2), 247-266.
- Meitz M., Preve D., Saikkonen P. 2023. A mixture autoregressive model based on Student's t-distribution. *Communications in Statistics Theory and Methods*, **52**(2), 499-515.
- Virolainen S. 2022. A mixture autoregressive model based on Gaussian and Student's t-distributions. Studies in Nonlinear Dynamics & Econometrics, **26**(4) 559-580.

# See Also

profile\_logliks, fitGSMAR, GSMAR, diagnostic\_plot, predict.gsmar, get\_test\_Omega,

46 predict.gsmar

# **Examples**

predict.gsmar

Forecast GMAR, StMAR, or G-StMAR process

## **Description**

predict.gsmar forecasts the specified GMAR, StMAR, or G-StMAR process by using the given data to simulate its possible future values. For one-step forecasts using the exact formula for conditional mean is supported.

# Usage

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# Arguments

object object of class 'gsmar' created with function fitGSMAR or GSMAR. additional arguments passed to grid (ignored if plot\_res==FALSE). a positive integer specifying how many steps in the future should be forecasted. n\_ahead nsimu a positive integer specifying to how many simulations the forecast should be based on. рi a numeric vector specifying confidence levels for the prediction intervals. should the prediction be based on sample "median" or "mean"? Or should it be pred\_type one-step-ahead forecast based on the exact conditional mean ("cond\_mean")? prediction intervals won't be calculated if the exact conditional mean is used. should the prediction intervals be "two-sided", "upper", or "lower"? pi\_type plot\_res a logical argument defining whether the forecast should be plotted or not. mix\_weights TRUE if forecasts for mixing weights should be plotted, FALSE in not. nt a positive integer specifying the number of observations to be plotted along with the prediction. Default is round(length(data)\*0.15).

#### **Details**

predict.gsmar uses the last p values of the data to simulate nsimu possible future values for each step-ahead. The point prediction is then obtained by calculating the sample median or mean for each step and the prediction intervals are obtained from the empirical fractiles.

The function simulate.gsmar can also be used directly for quantile based forecasting.

## Value

Returns a class 'gsmarpred' object containing, among the specifications,...

\$pred Point forecasts
\$pred\_ints Prediction intervals

\$mix\_pred Point forecasts for mixing weights

mix\_pred\_ints Individual prediction intervals for mixing weights, as [, , m], m=1,..,M.

#### References

- Galbraith, R., Galbraith, J. 1974. On the inverses of some patterned matrices arising in the theory of stationary time series. *Journal of Applied Probability* **11**, 63-71.
- Kalliovirta L. (2012) Misspecification tests based on quantile residuals. *The Econometrics Journal*, **15**, 358-393.
- Kalliovirta L., Meitz M. and Saikkonen P. 2015. Gaussian Mixture Autoregressive model for univariate time series. *Journal of Time Series Analysis*, **36**(2), 247-266.
- Meitz M., Preve D., Saikkonen P. 2023. A mixture autoregressive model based on Student's t-distribution. *Communications in Statistics Theory and Methods*, **52**(2), 499-515.
- Virolainen S. 2022. A mixture autoregressive model based on Gaussian and Student's t-distributions. Studies in Nonlinear Dynamics & Econometrics, 26(4) 559-580.

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## See Also

simulate.gsmar,cond\_moments,fitGSMAR,GSMAR,quantile\_residual\_tests,diagnostic\_plot

# **Examples**

```
## These examples take approximately 30 seconds to run.
# G-StMAR model with one GMAR type and one StMAR type regime
fit42gs <- fitGSMAR(M10Y1Y, p=4, M=c(1, 1), model="G-StMAR",
                    ncalls=1, seeds=4)
# Forecast 12 steps ahead based on 10000 simulated sample paths, prediction
# interval confidence levels 0.95 and 0.8, prediction based on sample median,
# and two-sided prediction intevals:
mypred <- predict(fit42gs, n_ahead=12, nsimu=10000, pi=c(0.95, 0.8),</pre>
                  pred_type="median", pi_type="two-sided")
mypred
plot(mypred)
# Forecast 24 steps ahead based on 1000 simulated sample paths, prediction
# interval confidence level 0.99 and 0.9, prediction based on sample mean,
# and upper prediction intevals:
mypred2 <- predict(fit42gs, n_ahead=24, nsimu=1000, pi=c(0.99, 0.9),</pre>
                   pred_type="mean", pi_type="upper")
# Forecast 24 steps ahead based on 1000 simulated sample paths, prediction
# interval confidence level 0.99, 0.95, 0.9 and 0.8, prediction based on
# sample median, and lower prediction intevals:
mypred3 <- predict(fit42gs, n_ahead=24, nsimu=1000, pi=c(0.99, 0.95, 0.9, 0.8),
                   pred_type="median", pi_type="lower")
# GMAR model
params12 <- c(1.70, 0.85, 0.30, 4.12, 0.73, 1.98, 0.63)
gmar12 <- GSMAR(data=simudata, p=1, M=2, params=params12, model="GMAR")</pre>
pred12 <- predict(gmar12, n_ahead=10, nsimu=1000, pi=c(0.95, 0.9, 0.8),</pre>
                  pred_type="median", pi_type="two-sided")
pred12
plot(pred12)
# One-step prediction based on the exact conditional mean:
predict(gmar12, n_ahead=1, pred_type="cond_mean", plot_res=FALSE)
```

print.gsmarpred

Print method for class 'gsmarpred' objects

# **Description**

print.gsmarpred is a print method for call 'gsmarpred' objects created with predict.gsmar.

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# Usage

```
## S3 method for class 'gsmarpred'
print(x, ..., digits = 2)
```

# Arguments

x object of class 'gsmarpred' generated by predict.gsmar.

... currently not in use.

digits the number of digits to be printed

print.gsmarsum

Print method from objects of class 'gsmarsum'

# Description

print.gsmarsum is a print method for objects of class 'gsmarsum' created with the summary method summary.gsmar. Approximate standard errors are printed in brackets.

# Usage

```
## S3 method for class 'gsmarsum'
print(x, ..., digits)
```

## **Arguments**

x object of class 'gsmarsum' generated by summary.gsmar.

... currently not in use.

digits the number of digits to be printed

profile\_logliks

Plot profile log-likelihoods around the estimates

## **Description**

profile\_logliks plots profile log-likelihoods around the estimates.

## Usage

```
profile_logliks(gsmar, scale = 0.02, nrows, ncols, precision = 200)
```

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## Arguments

gsmar	a class 'gsmar' object, typically generated by fitGSMAR or GSMAR.
scale	a numeric scalar specifying the interval plotted for each estimate: the estimate plus-minus abs(scale*estimate).
nrows	how many rows should be in the plot-matrix? The default is max(ceiling(log2(nparams) - 1), 1).
ncols	how many columns should be in the plot-matrix? The default is ceiling(nparams/nrows). Note that nrows*ncols should not be smaller than the number of parameters.
precision	at how many points should each profile log-likelihood be evaluated at?

#### **Details**

The red vertical line points the estimate.

Be aware that the profile log-likelihood function is subject to a numerical error due to limited float-point precision when considering extremely large parameter values, say, overly large degrees freedom estimates.

#### Value

Only plots to a graphical device and doesn't return anything.

## References

- Galbraith, R., Galbraith, J. 1974. On the inverses of some patterned matrices arising in the theory of stationary time series. *Journal of Applied Probability* **11**, 63-71.
- Kalliovirta L. (2012) Misspecification tests based on quantile residuals. *The Econometrics Journal*, **15**, 358-393.
- Kalliovirta L., Meitz M. and Saikkonen P. 2015. Gaussian Mixture Autoregressive model for univariate time series. *Journal of Time Series Analysis*, **36**(2), 247-266.
- Meitz M., Preve D., Saikkonen P. 2023. A mixture autoregressive model based on Student's t-distribution. *Communications in Statistics Theory and Methods*, **52**(2), 499-515.
- Virolainen S. 2022. A mixture autoregressive model based on Gaussian and Student's t-distributions. Studies in Nonlinear Dynamics & Econometrics, 26(4) 559-580.

## See Also

```
quantile_residual_plot, diagnostic_plot, cond_moment_plot, GSMAR, quantile_residual_tests,
simulate.gsmar
```

# **Examples**

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```
# GMAR model, graphs zoomed in closer.
fit12 <- fitGSMAR(data=simudata, p=1, M=2, model="GMAR", ncalls=1, seeds=1)
profile_logliks(fit12, scale=0.001)</pre>
```

quantile\_residuals

Compute quantile residuals of GMAR, StMAR, or G-StMAR model

## **Description**

quantile\_residuals computes the quantile residuals of the specified GMAR, StMAR, or G-StMAR model.

# Usage

```
quantile_residuals(
  data,
  p,
  M,
  params,
  model = c("GMAR", "StMAR", "G-StMAR"),
  restricted = FALSE,
  constraints = NULL,
  parametrization = c("intercept", "mean")
)
```

## **Arguments**

data

a numeric vector or class 'ts' object containing the data. NA values are not supported.

р

a positive integer specifying the autoregressive order of the model.

М

**For GMAR and StMAR models:** a positive integer specifying the number of mixture components.

For G-StMAR models: a size  $(2 \times 1)$  integer vector specifying the number of *GMAR type* components M1 in the first element and *StMAR type* components M2 in the second element. The total number of mixture components is M=M1+M2.

params

a real valued parameter vector specifying the model.

For non-restricted models: Size  $(M(p+3)+M-M1-1\times 1)$  vector  $\theta=(v_1,...,v_M,\alpha_1,...,\alpha_{M-1},\nu)$  where

```
• v_m = (\phi_{m,0}, \phi_m, \sigma_m^2)
```

• 
$$\phi_m = (\phi_{m,1}, ..., \phi_{m,p}), m = 1, ..., M$$

- $\nu = (\nu_{M1+1}, ..., \nu_M)$
- M1 is the number of GMAR type regimes.

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In the GMAR model, M1=M and the parameter  $\nu$  dropped. In the StMAR model, M1=0.

If the model imposes **linear constraints** on the autoregressive parameters: Replace the vectors  $\phi_m$  with the vectors  $\psi_m$  that satisfy  $\phi_m = C_m \psi_m$  (see the argument constraints).

For restricted models: Size  $(3M+M-M1+p-1\times1)$  vector  $\theta=(\phi_{1,0},...,\phi_{M,0},\phi,\sigma_1^2,...,\sigma_M^2,\alpha_1,...,\alpha_{M-1},\nu)$ , where  $\phi=(\phi_1,...,\phi_p)$  contains the AR coefficients, which are common for all regimes.

If the model imposes **linear constraints** on the autoregressive parameters: Replace the vector  $\phi$  with the vector  $\psi$  that satisfies  $\phi{=}C\psi$  (see the argument constraints).

Symbol  $\phi$  denotes an AR coefficient,  $\sigma^2$  a variance,  $\alpha$  a mixing weight, and  $\nu$  a degrees of freedom parameter. If parametrization=="mean", just replace each intercept term  $\phi_{m,0}$  with the regimewise mean  $\mu_m = \phi_{m,0}/(1-\sum\phi_{i,m})$ . In the **G-StMAR** model, the first M1 components are *GMAR type* and the rest M2 components are *StMAR type*. Note that in the case **M=1**, the mixing weight parameters  $\alpha$  are dropped, and in the case of **StMAR** or **G-StMAR** model, the degrees of freedom parameters  $\nu$  have to be larger than 2.

mode1

is "GMAR", "StMAR", or "G-StMAR" model considered? In the G-StMAR model, the first M1 components are *GMAR type* and the rest M2 components are *StMAR type*.

restricted

a logical argument stating whether the AR coefficients  $\phi_{m,1},...,\phi_{m,p}$  are restricted to be the same for all regimes.

constraints

specifies linear constraints imposed to each regime's autoregressive parameters separately.

For non-restricted models: a list of size  $(p \times q_m)$  constraint matrices  $C_m$  of full column rank satisfying  $\phi_m = C_m \psi_m$  for all m = 1, ..., M, where  $\phi_m = (\phi_{m,1}, ..., \phi_{m,p})$  and  $\psi_m = (\psi_{m,1}, ..., \psi_{m,q_m})$ .

**For restricted models:** a size  $(p \times q)$  constraint matrix C of full column rank satisfying  $\phi = C\psi$ , where  $\phi = (\phi_1, ..., \phi_p)$  and  $\psi = \psi_1, ..., \psi_q$ .

The symbol  $\phi$  denotes an AR coefficient. Note that regardless of any constraints, the autoregressive order is always p for all regimes. Ignore or set to NULL if applying linear constraints is not desired.

parametrization

is the model parametrized with the "intercepts"  $\phi_{m,0}$  or "means"  $\mu_m = \phi_{m,0}/(1-\sum \phi_{i,m})$ ?

# **Details**

Numerical integration is employed if the quantile residuals cannot be obtained analytically with the hypergeometric function using the package 'gsl'.

#### Value

Returns a (Tx1) numeric vector containing the quantile residuals of the specified GMAR, StMAR or G-StMAR model. Note that there are no quantile residuals for the first p observations as they are the initial values.

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## References

• Galbraith, R., Galbraith, J. 1974. On the inverses of some patterned matrices arising in the theory of stationary time series. *Journal of Applied Probability* **11**, 63-71.

- Kalliovirta L. (2012) Misspecification tests based on quantile residuals. *The Econometrics Journal*, **15**, 358-393.
- Kalliovirta L., Meitz M. and Saikkonen P. 2015. Gaussian Mixture Autoregressive model for univariate time series. *Journal of Time Series Analysis*, **36**(2), 247-266.
- Meitz M., Preve D., Saikkonen P. 2023. A mixture autoregressive model based on Student's t-distribution. *Communications in Statistics Theory and Methods*, **52**(2), 499-515.
- Virolainen S. 2022. A mixture autoregressive model based on Gaussian and Student's t-distributions. Studies in Nonlinear Dynamics & Econometrics, 26(4) 559-580.

# Examples

quantile\_residual\_plot

Plot quantile residual time series and histogram

## **Description**

quantile\_residualsPlot plots quantile residual time series and histogram.

# Usage

```
quantile_residual_plot(gsmar)
```

## **Arguments**

gsmar

a class 'gsmar' object, typically generated by fitGSMAR or GSMAR.

#### Value

Only plots to a graphical device and doesn't return anything.

## References

• Galbraith, R., Galbraith, J. 1974. On the inverses of some patterned matrices arising in the theory of stationary time series. *Journal of Applied Probability* **11**, 63-71.

- Kalliovirta L. (2012) Misspecification tests based on quantile residuals. *The Econometrics Journal*, **15**, 358-393.
- Kalliovirta L., Meitz M. and Saikkonen P. 2015. Gaussian Mixture Autoregressive model for univariate time series. *Journal of Time Series Analysis*, **36**(2), 247-266.
- Meitz M., Preve D., Saikkonen P. 2023. A mixture autoregressive model based on Student's t-distribution. *Communications in Statistics Theory and Methods*, **52**(2), 499-515.
- Virolainen S. 2022. A mixture autoregressive model based on Gaussian and Student's t-distributions. Studies in Nonlinear Dynamics & Econometrics, 26(4) 559-580.

#### See Also

```
profile_logliks, diagnostic_plot, fitGSMAR, GSMAR, quantile_residual_tests, simulate.gsmar
```

## **Examples**

random\_ind

Create random GMAR, StMAR, or G-StMAR model compatible parameter vector

## **Description**

random\_ind creates a random GMAR, StMAR, or G-StMAR model compatible mean-parametrized parameter vector.

smart\_ind creates a random GMAR, StMAR, or G-StMAR model compatible parameter vector close to argument params. Sometimes returns exactly the given parameter vector.

# Usage

```
random_ind(
  p,
 Μ,
 model = c("GMAR", "StMAR", "G-StMAR"),
  restricted = FALSE,
  constraints = NULL,
 mu_scale,
  sigma_scale,
  forcestat = FALSE
smart_ind(
  p,
 Μ,
  params,
 model = c("GMAR", "StMAR", "G-StMAR"),
  restricted = FALSE,
  constraints = NULL,
 mu_scale,
  sigma_scale,
  accuracy,
 which_random = numeric(0),
  forcestat = FALSE
)
```

## Arguments

p a positive integer specifying the autoregressive order of the model.

**For GMAR and StMAR models:** a positive integer specifying the number of mixture components.

For G-StMAR models: a size  $(2 \times 1)$  integer vector specifying the number of *GMAR type* components M1 in the first element and *StMAR type* components M2 in the second element. The total number of mixture components in M-M1+M2

is "GMAR", "StMAR", or "G-StMAR" model considered? In the G-StMAR model, the first M1 components are *GMAR type* and the rest M2 components are *StMAR type*.

a logical argument stating whether the AR coefficients  $\phi_{m,1},...,\phi_{m,p}$  are restricted to be the same for all regimes.

specifies linear constraints imposed to each regime's autoregressive parameters separately.

For non-restricted models: a list of size  $(p \times q_m)$  constraint matrices  $C_m$  of full column rank satisfying  $\phi_m = C_m \psi_m$  for all m = 1, ..., M, where  $\phi_m = (\phi_{m,1}, ..., \phi_{m,p})$  and  $\psi_m = (\psi_{m,1}, ..., \psi_{m,q_m})$ .

For restricted models: a size  $(p \times q)$  constraint matrix C of full column rank satisfying  $\phi = C\psi$ , where  $\phi = (\phi_1, ..., \phi_p)$  and  $\psi = \psi_1, ..., \psi_q$ .

0

М

model

restricted

constraints

> The symbol  $\phi$  denotes an AR coefficient. Note that regardless of any constraints, the autoregressive order is always p for all regimes. Ignore or set to NULL if applying linear constraints is not desired.

mu\_scale

a real valued vector of length two specifying the mean (the first element) and standard deviation (the second element) of the normal distribution from which the  $\phi_{m,0}$  or  $\mu_m$  (depending on the desired parametrization) parameters (for random regimes) should be generated.

sigma\_scale

a positive real number specifying the standard deviation of the (zero mean, positive only by taking absolute value) normal distribution from which the component variance parameters (for random regimes) should be generated.

forcestat

use the algorithm by Monahan (1984) to force stationarity on the AR parameters (slower) for random regimes? Not supported for constrained models.

params

a real valued parameter vector specifying the model.

For non-restricted models: Size  $(M(p+3)+M-M1-1\times 1)$  vector  $\theta=(v_1,...,v_M,$  $\alpha_1, ..., \alpha_{M-1}, \nu$ ) where

- $v_m = (\phi_{m,0}, \phi_m, \sigma_m^2)$
- $\phi_m = (\phi_{m,1}, ..., \phi_{m,p}), m = 1, ..., M$
- $\nu = (\nu_{M1+1}, ..., \nu_M)$
- M1 is the number of GMAR type regimes.

In the **GMAR** model, M1 = M and the parameter  $\nu$  dropped. In the **StMAR** model, M1 = 0.

If the model imposes **linear constraints** on the autoregressive parameters: Replace the vectors  $\phi_m$  with the vectors  $\psi_m$  that satisfy  $\phi_m = C_m \psi_m$  (see the argument constraints).

 $\sigma_1^2,...,\sigma_M^2,\alpha_1,...,\alpha_{M-1},\nu)$ , where  $\phi$ = $(\phi_1,...,\phi_p)$  contains the AR coefficients, which are common for all regimes.

If the model imposes **linear constraints** on the autoregressive parameters: Replace the vector  $\phi$  with the vector  $\psi$  that satisfies  $\phi = C\psi$  (see the argument constraints).

Symbol  $\phi$  denotes an AR coefficient,  $\sigma^2$  a variance,  $\alpha$  a mixing weight, and  $\nu$ a degrees of freedom parameter. If parametrization=="mean", just replace each intercept term  $\phi_{m,0}$  with the regimewise mean  $\mu_m = \phi_{m,0}/(1-\sum \phi_{i,m})$ . In the **G-StMAR** model, the first M1 components are *GMAR type* and the rest M2 components are StMAR type. Note that in the case M=1, the mixing weight parameters  $\alpha$  are dropped, and in the case of **StMAR** or **G-StMAR** model, the degrees of freedom parameters  $\nu$  have to be larger than 2.

accuracy

a real number larger than zero specifying how close to params the generated parameter vector should be. Standard deviation of the normal distribution from which new parameter values are drawn from will be corresponding parameter value divided by accuracy.

which\_random

a numeric vector of maximum length M specifying which regimes should be random instead of "smart" when using smart\_ind. Does not affect mixing weight parameters. Default in none.

#### **Details**

These functions can be used, for example, to create initial populations for the genetic algorithm. Mean-parametrization (instead of intercept terms  $\phi_{m,0}$ ) is assumed.

#### Value

Returns estimated parameter vector with the form described in initpop.

#### References

 Monahan J.F. 1984. A Note on Enforcing Stationarity in Autoregressive-Moving Average Models. *Biometrica* 71, 403-404.

## **Examples**

```
set.seed(1)
# GMAR model parameter vector
params22 <- random_ind(p=2, M=2, mu_scale=c(0, 1), sigma_scale=1)
smart22 <- smart_ind(p=2, M=2, params22, accuracy=10)</pre>
cbind(params22, smart22)
# Restricted GMAR parameter vector
params12r <- random_ind(p=1, M=2, restricted=TRUE, mu_scale=c(-2, 2), sigma_scale=2)</pre>
smart12r <- smart_ind(p=1, M=2, params12r, restricted=TRUE, accuracy=20)</pre>
cbind(params12r, smart12r)
# StMAR parameter vector: first regime is random in the "smart individual"
params13t <- random_ind(p=1, M=3, model="StMAR", mu_scale=c(3, 1), sigma_scale=3)</pre>
smart13t <- smart_ind(p=1, M=3, params13t, model="StMAR", accuracy=15,</pre>
                      mu_scale=c(3, 3), sigma_scale=3, which_random=1)
cbind(params13t, smart13t)
# Restricted StMAR parameter vector
params22tr <- random_ind(p=2, M=2, model="StMAR", restricted=TRUE,</pre>
                         mu_scale=c(3, 2), sigma_scale=0.5)
smart22tr <- smart_ind(p=2, M=2, params22tr, model="StMAR", restricted=TRUE,</pre>
                        accuracy=30)
cbind(params22tr, smart22tr)
# G-StMAR parameter vector
params12gs < random_ind(p=1, M=c(1, 1), model="G-StMAR", mu_scale=c(0, 1),
                          sigma_scale=1)
smart12gs <- smart_ind(p=1, M=c(1, 1), params12gs, model="G-StMAR",</pre>
                        accuracy=20)
cbind(params12gs, smart12gs)
# Such StMAR(3,2) that the AR coefficients are restricted to be
# the same for both regimes and that the second AR coefficients are
# constrained to zero. Second regime is random in the "smart individual".
params32trc <- random_ind(p=3, M=2, model="StMAR", restricted=TRUE,</pre>
                           constraints=matrix(c(1, 0, 0, 0, 0, 1), ncol=2),
```

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```
\label{eq:mu_scale} mu\_scale=c(-2,\ 0.5),\ sigma\_scale=4) $$ mart32trc <- \ smart\_ind(p=3,\ M=2,\ params32trc,\ model="StMAR",\ restricted=TRUE, constraints=matrix(c(1,\ 0,\ 0,\ 0,\ 0,\ 1),\ ncol=2), $$ mu\_scale=c(0,\ 0.1),\ sigma\_scale=0.1,\ which\_random=2, accuracy=20) $$ cbind(params32trc,\ smart32trc)
```

reform\_parameters

Reform any parameter vector into standard form.

# Description

reform\_parameters takes a parameter vector of any (non-constrained) GMAR, StMAR, or G-StMAR model and returns a list with the parameter vector in the standard form, parameter matrix containing AR coefficients and component variances, mixing weights alphas, and in case of StMAR or G-StMAR model also degrees of freedom parameters.

# Usage

```
reform_parameters(
  p,
  M,
  params,
  model = c("GMAR", "StMAR", "G-StMAR"),
  restricted = FALSE
)
```

# **Arguments**

p a positive integer specifying the autoregressive order of the model.

M **For GMAR and StMAR models:** a positive integer specifying the number of mixture components.

For G-StMAR models: a size  $(2 \times 1)$  integer vector specifying the number of *GMAR type* components M1 in the first element and *StMAR type* components M2 in the second element. The total number of mixture components is M=M1+M2.

params

a real valued parameter vector specifying the model.

For non-restricted models: Size  $(M(p+3)+M-M1-1\times1)$  vector  $\theta=(v_1,...,v_M,$ 

```
\alpha_1, ..., \alpha_{M-1}, \nu) where
```

- $v_m = (\phi_{m,0}, \phi_m, \sigma_m^2)$
- $\phi_m = (\phi_{m,1}, ..., \phi_{m,p}), m = 1, ..., M$
- $\nu = (\nu_{M1+1}, ..., \nu_M)$
- M1 is the number of GMAR type regimes.

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In the GMAR model, M1 = M and the parameter  $\nu$  dropped. In the StMAR model, M1 = 0.

If the model imposes **linear constraints** on the autoregressive parameters: Replace the vectors  $\phi_m$  with the vectors  $\psi_m$  that satisfy  $\phi_m = C_m \psi_m$  (see the argument constraints).

For restricted models: Size  $(3M+M-M1+p-1\times1)$  vector  $\theta=(\phi_{1,0},...,\phi_{M,0},\phi,\sigma_1^2,...,\sigma_M^2,\alpha_1,...,\alpha_{M-1},\nu)$ , where  $\phi=(\phi_1,...,\phi_p)$  contains the AR coefficients, which are common for all regimes.

If the model imposes **linear constraints** on the autoregressive parameters: Replace the vector  $\phi$  with the vector  $\psi$  that satisfies  $\phi{=}C\psi$  (see the argument constraints).

Symbol  $\phi$  denotes an AR coefficient,  $\sigma^2$  a variance,  $\alpha$  a mixing weight, and  $\nu$  a degrees of freedom parameter. If parametrization=="mean", just replace each intercept term  $\phi_{m,0}$  with the regimewise mean  $\mu_m = \phi_{m,0}/(1-\sum\phi_{i,m})$ . In the **G-StMAR** model, the first M1 components are *GMAR type* and the rest M2 components are *StMAR type*. Note that in the case **M=1**, the mixing weight parameters  $\alpha$  are dropped, and in the case of **StMAR** or **G-StMAR** model, the degrees of freedom parameters  $\nu$  have to be larger than 2.

model

is "GMAR", "StMAR", or "G-StMAR" model considered? In the G-StMAR model, the first M1 components are *GMAR type* and the rest M2 components are *StMAR type*.

restricted

a logical argument stating whether the AR coefficients  $\phi_{m,1},...,\phi_{m,p}$  are restricted to be the same for all regimes.

#### **Details**

This function does not support models imposing linear constraints. No argument checks in this function.

## Value

Returns a list with...

\$params parameter vector in the standard form.

\$pars corresponding parameter matrix containing AR coefficients and component variances. First row for phi0 or means depending on the parametrization. Column for each component.

\$alphas numeric vector containing mixing weight parameters for all of the components (also for the last one).

\$dfs numeric vector containing degrees of freedom parameters for all of components. Returned
only if model == "StMAR" or model == "G-StMAR".

@keywords internal

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simudata

Simulated data

# Description

A dataset containing 200 observations simulated from a GMAR p=1, M=2 process.

# Usage

simudata

## **Format**

A numeric vector of length 200.

# **Source**

Simulated

simulate.gsmar

Simulate obsercations from GMAR, StMAR, and G-StMAR processes

# **Description**

simulate.gsmar simulates observations from the specified GMAR, StMAR, or G-StMAR process. Can be utilized for forecasting future values of the process.

# Usage

```
## S3 method for class 'gsmar'
simulate(
  object,
  nsim = 1,
  seed = NULL,
  ...,
  init_values = NULL,
  ntimes = 1,
  drop = TRUE
)
```

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# Arguments

object	object of class 'gsmar', typically created with the function fitGSMAR or GSMAR.
nsim	a positive integer specifying how many values (ahead from init_values) will be simulated.
seed	an integer that specifies the seed for the random number generator. Ignored if $\ensuremath{NULL}$ .
	currently not in use.
init_values	a numeric vector with length >=p specifying the initial values for the simulation. The <b>last</b> element will be used as the initial value for the first lag, the second last element will be initial value for the second lag, etc. If NULL, initial values will be simulated from the process's stationary distribution.
ntimes	a positive integer specifying how many sets of simulations should be performed.
drop	if TRUE (default) then the components of the returned list are coerced to lower dimension if ntimes==1, i.e., \$sample and \$component will be vectors and \$mixing_weights will be matrix.

#### **Details**

The argument ntimes is intended for forecasting: a GSMAR process can be forecasted by simulating its possible future values. One can perform a large number of sets of simulations and calculate the sample quantiles from the simulated values to obtain prediction intervals. See the forecasting example below for a hand-on demonstration.

## Value

If drop==TRUE and ntimes==1 (default):  $sample and component are vectors and <math>mixing_weights$  is a  $(nsim \times M)$  matrix. Otherwise, returns a list with...

\$sample a size (nsim×ntimes) matrix containing the simulated values.

\$component a size (nsim×ntimes) matrix containing the information from which mixture component each value was generated from.

 $mixing_weights a size (nsim \times M \times ntimes)$  array containing the mixing weights corresponding to the sample: the dimension [i, , ] is the time index, the dimension [, i, ] indicates the regime, and the dimension [, , i] indicates the i:th set of simulations.

# References

- Galbraith, R., Galbraith, J. 1974. On the inverses of some patterned matrices arising in the theory of stationary time series. *Journal of Applied Probability* **11**, 63-71.
- Kalliovirta L. (2012) Misspecification tests based on quantile residuals. *The Econometrics Journal*, **15**, 358-393.
- Kalliovirta L., Meitz M. and Saikkonen P. 2015. Gaussian Mixture Autoregressive model for univariate time series. *Journal of Time Series Analysis*, 36(2), 247-266.
- Meitz M., Preve D., Saikkonen P. 2023. A mixture autoregressive model based on Student's t-distribution. *Communications in Statistics Theory and Methods*, **52**(2), 499-515.
- Virolainen S. 2022. A mixture autoregressive model based on Gaussian and Student's t-distributions. Studies in Nonlinear Dynamics & Econometrics, 26(4) 559-580.

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# See Also

fitGSMAR, GSMAR, predict.gsmar, add\_data, cond\_moments, mixing\_weights

# **Examples**

```
set.seed(1)
# GMAR model:
params22 <- c(0.9, 0.4, 0.2, 0.5, 0.7, 0.5, -0.2, 0.7, 0.7)
mod22 <- GSMAR(p=2, M=2, params=params22, model="GMAR")</pre>
mysim <- simulate(mod22, nsim=500)</pre>
ts.plot(mysim$sample)
ts.plot(mysim$component)
ts.plot(mysim$mixing_weights, col=rainbow(2), lty=2)
# G-StMAR model, with initial values:
params42gs <- c(0.04, 1.34, -0.59, 0.54, -0.36, 0.01, 0.06, 1.28, -0.36,
                0.2, -0.15, 0.04, 0.19, 9.75)
gstmar42 <- GSMAR(data=M10Y1Y, p=4, M=c(1, 1), params=params42gs,</pre>
                  model="G-StMAR")
sim42gs <- simulate(gstmar42, nsim=500, init_values=1:4)</pre>
ts.plot(sim42gs$sample)
ts.plot(sim42gs$component)
ts.plot(sim42gs$mixing_weights, col=rainbow(2), lty=2)
# FORECASTING EXAMPLE:
# GMAR model, 1000 sets of simulations with initial values from the data:
params12 <- c(1.70, 0.85, 0.30, 4.12, 0.73, 1.98, 0.63)
gmar12 <- GSMAR(data=simudata, p=1, M=2, params=params12, model="GMAR")</pre>
sim12 <- simulate(gmar12, nsim=5, init_val=gmar12$data, ntimes=1000)</pre>
apply(sim12$sample, MARGIN=1, FUN=median) # Point prediction
apply(sim12$sample, MARGIN=1, FUN=quantile, probs=c(0.025, 0.975)) # 95% pi
apply(sim12$mixing_weights, MARGIN=1:2, FUN=median) # mix.weight point pred
apply(sim12$mixing_weights, MARGIN=1:2, FUN=quantile,
      probs=c(0.025, 0.975)) # mix.weight 95% prediction intervals
```

stmarpars\_to\_gstmar

Transform a StMAR or G-StMAR model parameter vector to a corresponding G-StMAR model parameter vector with large dfs parameters reduced.

# **Description**

stmarpars\_to\_gstmar transforms a StMAR model parameter vector to a corresponding G-StMAR model parameter vector with large dfs parameters reduced by switching the related regimes to be GMAR type.

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## Usage

```
stmarpars_to_gstmar(
  p,
  M,
  params,
  model = c("GMAR", "StMAR", "G-StMAR"),
  restricted = FALSE,
  constraints = NULL,
  maxdf = 100
)
```

#### **Arguments**

р

a positive integer specifying the autoregressive order of the model.

М

**For GMAR and StMAR models:** a positive integer specifying the number of mixture components.

For G-StMAR models: a size  $(2 \times 1)$  integer vector specifying the number of *GMAR type* components M1 in the first element and *StMAR type* components M2 in the second element. The total number of mixture components is M=M1+M2.

params

a real valued parameter vector specifying the model.

 $\alpha_1,...,\alpha_{M-1},\nu)$  where

- $v_m = (\phi_{m,0}, \phi_m, \sigma_m^2)$
- $\phi_m = (\phi_{m,1}, ..., \phi_{m,p}), m = 1, ..., M$
- $\nu = (\nu_{M1+1}, ..., \nu_M)$
- M1 is the number of GMAR type regimes.

In the GMAR model, M1=M and the parameter  $\nu$  dropped. In the StMAR model, M1=0.

If the model imposes **linear constraints** on the autoregressive parameters: Replace the vectors  $\phi_m$  with the vectors  $\psi_m$  that satisfy  $\phi_m = C_m \psi_m$  (see the argument constraints).

For restricted models: Size  $(3M+M-M1+p-1\times1)$  vector  $\theta=(\phi_{1,0},...,\phi_{M,0},\phi,\sigma_1^2,...,\sigma_M^2,\alpha_1,...,\alpha_{M-1},\nu)$ , where  $\phi=(\phi_1,...,\phi_p)$  contains the AR coefficients, which are common for all regimes.

If the model imposes **linear constraints** on the autoregressive parameters: Replace the vector  $\phi$  with the vector  $\psi$  that satisfies  $\phi{=}C\psi$  (see the argument constraints).

Symbol  $\phi$  denotes an AR coefficient,  $\sigma^2$  a variance,  $\alpha$  a mixing weight, and  $\nu$  a degrees of freedom parameter. If parametrization=="mean", just replace each intercept term  $\phi_{m,0}$  with the regimewise mean  $\mu_m = \phi_{m,0}/(1-\sum\phi_{i,m})$ . In the **G-StMAR** model, the first M1 components are *GMAR type* and the rest M2 components are *StMAR type*. Note that in the case **M=1**, the mixing weight parameters  $\alpha$  are dropped, and in the case of **StMAR** or **G-StMAR** model, the degrees of freedom parameters  $\nu$  have to be larger than 2.

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model is "GMAR", "StMAR", or "G-StMAR" model considered? In the G-StMAR

model, the first M1 components are  $\emph{GMAR}$   $\emph{type}$  and the rest M2 components are

StMAR type.

restricted a logical argument stating whether the AR coefficients  $\phi_{m,1},...,\phi_{m,p}$  are re-

stricted to be the same for all regimes.

constraints specifies linear constraints imposed to each regime's autoregressive parameters

separately.

For non-restricted models: a list of size  $(p \times q_m)$  constraint matrices  $C_m$  of full column rank satisfying  $\phi_m = C_m \psi_m$  for all m = 1, ..., M, where  $\phi_m = (\phi_{m,1}, ..., \phi_{m,p})$  and  $\psi_m = (\psi_{m,1}, ..., \psi_{m,q_m})$ .

**For restricted models:** a size  $(p \times q)$  constraint matrix C of full column rank satisfying  $\phi = C\psi$ , where  $\phi = (\phi_1, ..., \phi_p)$  and  $\psi = \psi_1, ..., \psi_q$ .

The symbol  $\phi$  denotes an AR coefficient. Note that regardless of any constraints, the autoregressive order is always p for all regimes. Ignore or set to NULL if applying linear constraints is not desired.

maxdf regimes with degrees of freedom parameter value larger than this will be turned

into GMAR type.

#### Value

Returns a list with three elements: \$params contains the corresponding G-StMAR model parameter vector, \$reg\_order contains the permutation that was applied to the regimes (GMAR type components first, and decreasing ordering by mixing weight parameters), and \$M a vector of length two containing the number of GMAR type regimes in the first element and the number of StMAR type regimes in the second.

## **Examples**

```
params12 <- c(2, 0.9, 0.1, 0.8, 0.5, 0.5, 0.4, 12, 300)
stmarpars_to_gstmar(p=1, M=2, params=params12, model="StMAR", maxdf=100)
```

stmar\_to\_gstmar

Estimate a G-StMAR model based on a StMAR model with large degrees of freedom parameters

# Description

stmar\_to\_gstmar estimates a G-StMAR model based on a StMAR model with large degree of freedom parameters.

## Usage

```
stmar_to_gstmar(
  gsmar,
  maxdf = 100,
  estimate,
```

stmar\_to\_gstmar 65

```
calc_std_errors,
maxit = 100,
custom_h = NULL
)
```

#### **Arguments**

gsmar a class 'gsmar' object, typically generated by fitGSMAR or GSMAR.

maxdf regimes with degrees of freedom parameter value larger than this will be turned

into GMAR type.

estimate set TRUE if the new model should be estimated with a variable metric algorithm

using the StMAR model parameter value as the initial value. By default TRUE iff

the model contains data.

calc\_std\_errors

set TRUE if the approximate standard errors should be calculated. By default

TRUE iff the model contains data.

maxit the maximum number of iterations for the variable metric algorithm. Ignored if

estimate==FALSE.

custom\_h A numeric vector with same the length as the parameter vector: i:th element of

custom\_h is the difference used in central difference approximation for partial differentials of the log-likelihood function for the i:th parameter. If NULL (default), then the difference used for differentiating overly large degrees of freedom parameters is adjusted to avoid numerical problems, and the difference is

6e-6 for the other parameters.

#### **Details**

If a StMAR model contains large estimates for the degrees of freedom parameters, one should consider switching to the corresponding G-StMAR model that lets the corresponding regimes to be GMAR type. stmar\_to\_gstmar does this switch conveniently.

# Value

Returns an object of class 'gsmar' defining the specified GMAR, StMAR, or G-StMAR model. If data is supplied, the returned object contains (by default) empirical mixing weights, some conditional and unconditional moments, and quantile residuals. Note that the first p observations are taken as the initial values so the mixing weights, conditional moments, and quantile residuals start from the p+1:th observation (interpreted as t=1).

#### References

- Kalliovirta L., Meitz M. and Saikkonen P. 2015. Gaussian Mixture Autoregressive model for univariate time series. *Journal of Time Series Analysis*, 36(2), 247-266.
- Meitz M., Preve D., Saikkonen P. 2023. A mixture autoregressive model based on Student's t-distribution. *Communications in Statistics Theory and Methods*, **52**(2), 499-515.
- Virolainen S. 2022. A mixture autoregressive model based on Gaussian and Student's t-distributions. Studies in Nonlinear Dynamics & Econometrics, 26(4) 559-580.

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# See Also

```
fitGSMAR, GSMAR, iterate_more, get_gradient, get_regime_means, swap_parametrization,
stmar_to_gstmar
```

# **Examples**

```
# These are long running example that take approximately 15 seconds to run.
fit42t <- fitGSMAR(data=M10Y1Y, p=4, M=2, model="StMAR", ncalls=1, seeds=6)
fit42t # Overly large degrees of freedom estimate!

# Switch to the appropriate G-StMAR model:
fit42gs <- stmar_to_gstmar(fit42t)
fit42gs</pre>
```

swap\_parametrization Swap the parametrization of object of class 'gsmar' defining a GMAR, StMAR, or G-StMAR model

# Description

swap\_parametrization swaps the parametrization of object of class 'gsmar' to "mean" if the current parametrization is "intercept", and vice versa.

#### Usage

```
swap_parametrization(gsmar, calc_std_errors = TRUE, custom_h = NULL)
```

# Arguments

gsmar a class 'gsmar' object, typically generated by fitGSMAR or GSMAR. calc\_std\_errors

should approximate standard errors be calculated?

custom\_h A numeric vector with same the length as the param

A numeric vector with same the length as the parameter vector: i:th element of custom\_h is the difference used in central difference approximation for partial differentials of the log-likelihood function for the i:th parameter. If NULL (default), then the difference used for differentiating overly large degrees of freedom parameters is adjusted to avoid numerical problems, and the difference is 6e-6 for the other parameters.

# **Details**

swap\_parametrization is a convenient tool if you have estimated the model in "intercept"-parametrization but wish to work with "mean"-parametrization in the future, or vice versa. For example, approximate standard errors are readily available for parametrized parameters only.

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#### Value

Returns an object of class 'gsmar' defining the specified GMAR, StMAR, or G-StMAR model. If data is supplied, the returned object contains (by default) empirical mixing weights, some conditional and unconditional moments, and quantile residuals. Note that the first p observations are taken as the initial values so the mixing weights, conditional moments, and quantile residuals start from the p+1:th observation (interpreted as t=1).

#### References

- Kalliovirta L., Meitz M. and Saikkonen P. 2015. Gaussian Mixture Autoregressive model for univariate time series. *Journal of Time Series Analysis*, **36**(2), 247-266.
- Meitz M., Preve D., Saikkonen P. 2023. A mixture autoregressive model based on Student's t-distribution. *Communications in Statistics Theory and Methods*, **52**(2), 499-515.
- Virolainen S. 2022. A mixture autoregressive model based on Gaussian and Student's t-distributions. Studies in Nonlinear Dynamics & Econometrics, 26(4) 559-580.

#### See Also

```
fitGSMAR, GSMAR, iterate_more, get_gradient, get_regime_means, swap_parametrization,
stmar_to_gstmar
```

# **Examples**

T10Y1Y

Spread between 10-Year and 1-Year Treasury rates: T10Y1Y

## **Description**

A dataset containing monthly U.S. interest rate spread between the 10-Year Treasury constant maturity rate and 1-Year Treasury constant maturity rate from 1953IV to 2020II.

# Usage

T10Y1Y

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# **Format**

A class 'ts' time series object containing 803 observations.

# **Source**

https://fred.stlouisfed.org/series/GS10 https://fred.stlouisfed.org/series/GS1

**TBFF** 

Spread between the 3-month Treasury bill rate and the effective federal funds rate: TBFF

# **Description**

A dataset containing the monthly U.S. interest rate spread between the 3-month Treasury bill secondary market rate and the effective federal funds rate from 1954 July to 2019 July (781 observations). This series was studied in the empirical application of Virolainen (2021) introducing the G-StMAR model.

## Usage

**TBFF** 

# **Format**

A class 'ts' time series object containing 781 observations.

# **Source**

https://fred.stlouisfed.org/series/TB3SMFFM

# References

 Virolainen S. 2021. A mixture autoregressive model based on Gaussian and Student's tdistributions. Studies in Nonlinear Dynamics & Econometrics, doi: 10.1515/snde-2020-0060 uncond\_moments 69

uncond_moments	Calculate unconditional mean, variance, first p autocovariances and autocorrelations of the GSMAR process.

# **Description**

uncond\_moments calculates the unconditional mean, variance, and the first p autocovariances and autocorrelations of the GSMAR process.

#### Usage

```
uncond_moments(gsmar)
```

## **Arguments**

gsmar

a class 'gsmar' object, typically generated by fitGSMAR or GSMAR.

#### Value

Returns a list containing the unconditional mean, variance, and the first p autocovariances and autocorrelations. Note that the lag-zero autocovariance/correlation is not included in the "first p" but is given in the uncond\_variance component separately.

## References

- Kalliovirta L., Meitz M. and Saikkonen P. 2015. Gaussian Mixture Autoregressive model for univariate time series. *Journal of Time Series Analysis*, **36**(2), 247-266.
- Meitz M., Preve D., Saikkonen P. 2023. A mixture autoregressive model based on Student's t-distribution. *Communications in Statistics Theory and Methods*, **52**(2), 499-515.
- Virolainen S. 2022. A mixture autoregressive model based on Gaussian and Student's t-distributions. Studies in Nonlinear Dynamics & Econometrics, 26(4) 559-580.
- Lütkepohl H. 2005. New Introduction to Multiple Time Series Analysis. Springer.

#### See Also

Other moment functions: cond\_moments(), get\_regime\_autocovs(), get\_regime\_means(), get\_regime\_vars()

# **Examples**

```
# GMAR model
params13 <- c(1.4, 0.88, 0.26, 2.46, 0.82, 0.74, 5.0, 0.68, 5.2, 0.72, 0.2)
gmar13 <- GSMAR(p=1, M=3, params=params13, model="GMAR")
uncond_moments(gmar13)

# StMAR model
params12t <- c(1.38, 0.88, 0.27, 3.8, 0.74, 3.15, 0.8, 100, 3.6)
stmar12t <- GSMAR(p=1, M=2, params=params12t, model="StMAR")
uncond_moments(stmar12t)
```

70 Wald\_test

```
# G-StMAR model (similar to the StMAR model above)
params12gs <- c(1.38, 0.88, 0.27, 3.8, 0.74, 3.15, 0.8, 3.6)
gstmar12 <- GSMAR(p=1, M=c(1, 1), params=params12gs, model="G-StMAR")
uncond_moments(gstmar12)</pre>
```

Wald\_test

Perform Wald test

# **Description**

Wald\_test performs a Wald test for a GMAR, StMAR, or G-StMAR model.

# Usage

```
Wald_test(gsmar, A, c, h = 6e-06)
```

# **Arguments**

gsmar	a class 'gsmar' object, typically generated by fitGSMAR or GSMAR.
A	a size $(k \times n_p arams)$ matrix with full row rank specifying a part of the null hypothesis, where $n_p arams$ is the number of parameters in the (unconstrained) model. See details for more information.
С	a length $\boldsymbol{k}$ vector specifying a part of the null hypothesis. See details for more information.
h	the difference used to approximate the derivatives.

# Details

Denoting the true parameter value by  $\theta_0$ , we test the null hypothesis  $A\theta_0=c$ . Under the null, the test statistic is asymptotically  $\chi^2$ -distributed with k (=nrow(A)) degrees of freedom. The parameter  $\theta_0$  is assumed to have the same form as in the model supplied in the argument gsmar and it is presented in the documentation of the argument params in the function GSMAR (see ?GSMAR).

Note that this function does **not** check whether the specified constraints are feasible (e.g., whether the implied constrained model would be stationary or have positive definite error term covariance matrices).

#### Value

A list with class "htest" containing the following components:

statistic the value of the Wald statistics.

parameter the degrees of freedom of the Wald statistic.

p.value the p-value of the test.

alternative a character string describing the alternative hypothesis.

method a character string indicating the type of the test (Wald test).

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data.name	a character string giving the names of the supplied model, constraint matrix A, and vector c.
gsmar	the supplied argument gsmar.
Α	the supplied argument A.
С	the supplied argument c.
h	the supplied argument h.

#### References

- Kalliovirta L., Meitz M. and Saikkonen P. 2015. Gaussian Mixture Autoregressive model for univariate time series. *Journal of Time Series Analysis*, **36**(2), 247-266.
- Meitz M., Preve D., Saikkonen P. 2023. A mixture autoregressive model based on Student's t-distribution. *Communications in Statistics Theory and Methods*, **52**(2), 499-515.
- Virolainen S. 2022. A mixture autoregressive model based on Gaussian and Student's t-distributions. Studies in Nonlinear Dynamics & Econometrics, 26(4) 559-580.

## See Also

```
\label{local_local_local} LR\_test, \ fitGSMAR, \ GSMAR, \ diagnostic\_plot, \ profile\_logliks, \ quantile\_residual\_tests, \ cond\_moment\_plot
```

# **Examples**

```
# GMAR p=1, M=2 model:
fit12 <- fitGSMAR(simudata, p=1, M=2, model="GMAR", ncalls=1, seeds=1)
# Test with Wald test whether the AR coefficients are the same in both
# regimes:
# There are 7 parameters in the model and the AR coefficient of the
# first regime is the 2nd element, whereas the AR coefficient of the second
# regime is in the 5th element.
A <- matrix(c(0, 1, 0, 0, -1, 0, 0), nrow=1, ncol=7)
c <- 0
Wald_test(fit12, A=A, c=c)</pre>
```

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