# Package 'bnpsd'

July 22, 2025

Title Simulate Genotypes from the BN-PSD Admixture Model

**Version** 1.3.13

**Description** The Pritchard-Stephens-Donnelly (PSD) admixture model has k intermediate subpopulations from which n individuals draw their alleles dictated by their individual-specific admixture proportions. The BN-PSD model additionally imposes the Balding-Nichols (BN) allele frequency model to the intermediate populations, which therefore evolved independently from a common ancestral population T with subpopulation-specific FST (Wright's fixation index) parameters. The BN-PSD model can be used to yield complex population structures. This simulation approach is now extended to subpopulations related by a tree. Method described in Ochoa and Storey (2021) <doi:10.1371/journal.pgen.1009241>.

Imports stats, ape, nnls
Suggests popkin (>= 1.3.9), testthat, knitr, rmarkdown, RColorBrewer
License GPL (>= 3)
Encoding UTF-8
RoxygenNote 7.1.1
VignetteBuilder knitr
<pre>URL https://github.com/StoreyLab/bnpsd/</pre>
<pre>BugReports https://github.com/StoreyLab/bnpsd/issues</pre>
NeedsCompilation no
<b>Author</b> Alejandro Ochoa [aut, cre] (ORCID: <a href="https://orcid.org/0000-0003-4928-3403">https://orcid.org/0000-0003-4928-3403</a> ), John D. Storey [aut] (ORCID: <a href="https://orcid.org/0000-0001-5992-402X">https://orcid.org/0000-0001-5992-402X</a> )
Maintainer Alejandro Ochoa <alejandro.ochoa@duke.edu></alejandro.ochoa@duke.edu>
Repository CRAN
<b>Date/Publication</b> 2021-08-25 12:50:26 UTC

# **Contents**

admix_prop_1d_circular	•																		2
admix_prop_1d_linear																			2

```
Index
35
```

admix\_prop\_1d\_circular

Construct admixture proportion matrix for circular 1D geography

# Description

Assumes k\_subpops intermediate subpopulations placed along a circumference (the [0, 2 \* pi] line that wraps around) with even spacing spread by random walks (see details below), then n\_ind individuals sampled equally spaced in  $[coord_ind_first,coord_ind_last]$  (default [0, 2 \* pi] with a small gap so first and last individual do not overlap) draw their admixture proportions relative to the Von Mises density that models the random walks of each of these intermediate subpopulations. The spread of the random walks is sigma = 1 / sqrt(kappa) of the Von Mises density. If sigma is missing, it can be set indirectly by providing three variables: (1) the desired bias coefficient bias\_coeff, (2) the coancestry matrix of the intermediate subpopulations coanc\_subpops (up to a scalar factor), and (3) the final fst of the admixed individuals (see details below).

# Usage

```
admix_prop_1d_circular(
    n_ind,
    k_subpops,
    sigma = NA,
    coord_ind_first = 2 * pi/(2 * n_ind),
    coord_ind_last = 2 * pi * (1 - 1/(2 * n_ind)),
    bias_coeff = NA,
    coanc_subpops = NULL,
```

```
fst = NA
)
```

## **Arguments**

n\_ind Number of individuals

k\_subpops Number of intermediate subpopulations

sigma Spread of intermediate subpopulations (approximate standard deviation of Von

Mises densities, see above) The edge cases sigma = 0 and sigma = Inf are han-

dled appropriately!

coord\_ind\_first

Location of first individual

coord\_ind\_last Location of last individual

OPTIONS FOR BIAS COEFFICIENT VERSION

bias\_coeff If sigma is NA, this bias coefficient is required.

provided as a k\_subpops-by-k\_subpops matrix, a length-k\_subpops population inbreeding vector (for independent subpopulations, where between-subpop coancestries are zero) or scalar (if population inbreeding values are all equal and coancestries are zero). This coanc\_subpops can be in the wrong scale (it cancels out in calculations), which is returned corrected, to result in the desired fst

(next).

fst If sigma is NA, this FST of the admixed individuals is required.

### **Details**

Assuming the full range of [0, 2 \* pi] is considered, and the first and last individuals do not overlap, the gap between individuals is delta = 2 \* pi / n. To not have any individuals on the edge, we place the first individual at delta / 2 and the last at 2 \* pi - delta / 2. The location of subpopulation j is delta / 2 + (j - 1/2) / k \* (2 \* pi - delta), chosen to agree with the default correspondence between individuals and subpopulations of the linear 1D geography admixture scenario (admix\_prop\_1d\_linear()).

If sigma is NA, its value is determined from the desired bias\_coeff, coanc\_subpops up to a scalar factor, and fst. Uniform weights for the final generalized FST are assumed. The scale of coanc\_subpops is irrelevant because it cancels out in bias\_coeff; after sigma is found, coanc\_subpops is rescaled to give the desired final FST. However, the function stops if any rescaled coanc\_subpops values are greater than 1, which are not allowed since they are IBD probabilities.

#### Value

If sigma was provided, returns the n\_ind-by-k\_subpops admixture proportion matrix (admix\_proportions). If sigma is missing, returns a named list containing:

- admix\_proportions: the n\_ind-by-k\_subpops admixture proportion matrix. If coanc\_subpops had names, they are copied to the columns of this matrix.
- coanc\_subpops: the input coanc\_subpops rescaled.
- sigma: the fit value of the spread of intermediate subpopulations
- coanc\_factor: multiplicative factor used to rescale coanc\_subpops

# **Examples**

```
# admixture matrix for 1000 individuals drawing alleles from 10 subpops
# simple version: spread of about 2 standard deviations along the circular 1D geography
# (just set sigma)
admix_proportions <- admix_prop_1d_circular(n_ind = 1000, k_subpops = 10, sigma = 2)
# advanced version: a similar model but with a bias coefficient of exactly 1/2
# (must provide bias_coeff, coanc_subpops, and fst in lieu of sigma)
k_subpops <- 10
# FST vector for intermediate independent subpops, up to a factor (will be rescaled below)
coanc_subpops <- 1 : k_subpops</pre>
obj <- admix_prop_1d_circular(</pre>
   n_{ind} = 1000,
   k_{subpops} = k_{subpops}
   bias_coeff = 0.5,
    coanc_subpops = coanc_subpops,
    fst = 0.1 # desired final FST of admixed individuals
)
# in this case return value is a named list with three items:
admix_proportions <- obj$admix_proportions</pre>
# rescaled coancestry data (matrix or vector) for intermediate subpops
coanc_subpops <- obj$coanc_subpops</pre>
# and the sigma that gives the desired bias_coeff and final FST
sigma <- obj$sigma
```

admix\_prop\_1d\_linear Construct admixture proportion matrix for 1D geography

# Description

Assumes k\_subpops intermediate subpopulations placed along a line at locations 1 : k\_subpops spread by random walks, then n\_ind individuals equally spaced in [coord\_ind\_first,coord\_ind\_last] draw their admixture proportions relative to the Normal density that models the random walks of each of these intermediate subpopulations. The spread of the random walks (the standard deviation of the Normal densities) is sigma. If sigma is missing, it can be set indirectly by providing three variables: (1) the desired bias coefficient bias\_coeff, (2) the coancestry matrix of the intermediate subpopulations coanc\_subpops (up to a scalar factor), and (3) the final fst of the admixed individuals (see details below).

## Usage

```
admix_prop_1d_linear(
    n_ind,
    k_subpops,
    sigma = NA,
```

admix\_prop\_1d\_linear 5

```
coord_ind_first = 0.5,
coord_ind_last = k_subpops + 0.5,
bias_coeff = NA,
coanc_subpops = NULL,
fst = NA
)
```

#### **Arguments**

n\_ind Number of individuals.

k\_subpops Number of intermediate subpopulations.

sigma Spread of intermediate subpopulations (standard deviation of normal densities).

The edge cases sigma = 0 and sigma = Inf are handled appropriately!

coord\_ind\_first

Location of first individual (default 0.5).

coord\_ind\_last Location of last individual (default k\_subpops + 0.5).

OPTIONS FOR BIAS COEFFICIENT VERSION

bias\_coeff If sigma is NA, this bias coefficient is required.

provided as a k\_subpops-by-k\_subpops matrix, a length-k\_subpops population inbreeding vector (for independent subpopulations, where between-subpop coancestries are zero) or scalar (if population inbreeding values are all equal and coancestries are zero). This coanc\_subpops can be in the wrong scale (it cancels out in calculations), which is returned corrected, to result in the desired fst

(next).

fst If sigma is NA, this FST of the admixed individuals is required.

# **Details**

If sigma is NA, its value is determined from the desired bias\_coeff, coanc\_subpops up to a scalar factor, and fst. Uniform weights for the final generalized FST are assumed. The scale of coanc\_subpops is irrelevant because it cancels out in bias\_coeff; after sigma is found, coanc\_subpops is rescaled to give the desired final FST. However, the function stops if any rescaled coanc\_subpops values are greater than 1, which are not allowed since they are IBD probabilities.

# Value

If sigma was provided, returns the n\_ind-by-k\_subpops admixture proportion matrix (admix\_proportions). If sigma is missing, returns a named list containing:

- admix\_proportions: the n\_ind-by-k\_subpops admixture proportion matrix. If coanc\_subpops had names, they are copied to the columns of this matrix.
- coanc\_subpops: the input coanc\_subpops rescaled.
- sigma: the fit value of the spread of intermediate subpopulations
- coanc\_factor: multiplicative factor used to rescale coanc\_subpops

# **Examples**

```
# admixture matrix for 1000 individuals drawing alleles from 10 subpops
# simple version: spread of 2 standard deviations along the 1D geography
# (just set sigma)
admix_proportions <- admix_prop_1d_linear(n_ind = 1000, k_subpops = 10, sigma = 2)</pre>
# as sigma approaches zero, admix_proportions approaches the independent subpopulations matrix
admix_prop_1d_linear(n_ind = 10, k_subpops = 2, sigma = 0)
# advanced version: a similar model but with a bias coefficient of exactly 1/2
# (must provide bias_coeff, coanc_subpops, and fst in lieu of sigma)
k_subpops <- 10
# FST vector for intermediate independent subpops, up to a factor (will be rescaled below)
coanc_subpops <- 1 : k_subpops</pre>
obj <- admix_prop_1d_linear(</pre>
    n_{ind} = 1000,
   k_{subpops} = k_{subpops}
   bias_coeff = 0.5,
    coanc_subpops = coanc_subpops,
    fst = 0.1 # desired final FST of admixed individuals
)
# in this case return value is a named list with three items:
# admixture proportions
admix_proportions <- obj$admix_proportions</pre>
# rescaled coancestry data (matrix or vector) for intermediate subpops
coanc_subpops <- obj$coanc_subpops</pre>
# and the sigma that gives the desired bias_coeff and final FST
sigma <- obj$sigma
```

admix\_prop\_indep\_subpops

Construct admixture proportion matrix for independent subpopulations

#### **Description**

This function constructs an admixture proportion matrix where every individuals is actually unadmixed (draws its full ancestry from a single intermediate subpopulation). The inputs are the vector of subpopulation labels labs for every individual (length n), and the length-k vector of unique subpopulations subpops in the desired order. If subpops is missing, the sorted unique subpopulations observed in labs is used. This function returns the admixture proportion matrix, for each individual 1 for the column corresponding to its subpopulation, 0 otherwise.

# Usage

```
admix_prop_indep_subpops(labs, subpops = sort(unique(labs)))
```

bnpsd 7

# Arguments

labs Length-n vector of subpopulation labels

subpops Optional length-k vector of unique subpopulations in desired order. Stops if

subpops does not contain all unique labels in labs (no error if subpops contains

additional labels).

# Value

The n-by-k admixture proportion matrix. The unique subpopulation labels are given in the column names.

# **Examples**

```
# vector of subpopulation memberships
labs <- c(1, 1, 1, 2, 2, 3, 1)
# admixture matrix with subpopulations (along columns) sorted
admix_proportions <- admix_prop_indep_subpops(labs)

# declare subpopulations in custom order
subpops <- c(3, 1, 2)
# columns will be reordered to match subpops as provided
admix_proportions <- admix_prop_indep_subpops(labs, subpops)

# declare subpopulations with unobserved labels
subpops <- 1:5
# note columns 4 and 5 will be false for all individuals
admix_proportions <- admix_prop_indep_subpops(labs, subpops)</pre>
```

bnpsd

A package for modeling and simulating an admixed population

# **Description**

The underlying model is called the BN-PSD admixture model, which combines the Balding-Nichols (BN) allele frequency model for the intermediate subpopulations with the Pritchard-Stephens-Donnelly (PSD) model of individual-specific admixture proportions. The BN-PSD model enables the simulation of complex population structures, ideal for illustrating challenges in kinship coefficient and FST estimation. Simulated loci are drawn independently (in linkage equilibrium).

## Author(s)

Authors:

 $\begin{tabular}{ll} \textbf{Maintainer}: A lejandro Ochoa <a lejandro.ochoa@duke.edu> (ORCID) \\ \end{tabular}$ 

• John D. Storey <jstorey@princeton.edu> (ORCID)

8 bnpsd

# See Also

Useful links:

- https://github.com/StoreyLab/bnpsd/
- Report bugs at https://github.com/StoreyLab/bnpsd/issues

```
# dimensions of data/model
# number of loci
m loci <- 10
# number of individuals
n_{ind} < -5
# number of intermediate subpops
k_subpops < -2
# define population structure
\# FST values for k = 2 subpopulations
inbr\_subpops \leftarrow c(0.1, 0.3)
# admixture proportions from 1D geography
admix_proportions <- admix_prop_1d_linear( n_ind, k_subpops, sigma = 1 )</pre>
# also available:
# - admix_prop_1d_circular
# - admix_prop_indep_subpops
# get pop structure parameters of the admixed individuals
# the coancestry matrix
coancestry <- coanc_admix( admix_proportions, inbr_subpops )</pre>
# FST of admixed individuals
Fst <- fst_admix( admix_proportions, inbr_subpops )</pre>
# draw all random allele freqs and genotypes
out <- draw_all_admix( admix_proportions, inbr_subpops, m_loci )</pre>
# genotypes
X <- out$X
# ancestral allele frequencies (AFs)
p_anc <- out$p_anc</pre>
# OR... draw each vector or matrix separately
# provided for additional flexibility
# ancestral AFs
p_anc <- draw_p_anc( m_loci )</pre>
# independent subpops (intermediate) AFs
p_subpops <- draw_p_subpops( p_anc, inbr_subpops )</pre>
# individual-specific AFs
p_ind <- make_p_ind_admix( p_subpops, admix_proportions )</pre>
# genotypes
X <- draw_genotypes_admix( p_ind )</pre>
### Examples with a tree for intermediate subpopulations
# tree allows for correlated subpopulations
```

coanc\_admix 9

```
# (prev examples had independent subpopulations)
# best to start by specifying tree in Newick string format
tree_str <- '(S1:0.1,(S2:0.1,S3:0.1)N1:0.1)T;'
# and turn it into `phylo` object using the `ape` package
library(ape)
tree_subpops <- read.tree( text = tree_str )</pre>
# true coancestry matrix corresponding to this tree
coanc_subpops <- coanc_tree( tree_subpops )</pre>
# admixture proportions from 1D geography
# (constructed again but for k=3 tree)
k_subpops <- nrow( coanc_subpops )</pre>
admix_proportions <- admix_prop_1d_linear( n_ind, k_subpops, sigma = 0.5 )</pre>
# get pop structure parameters of the admixed individuals
# the coancestry matrix
coancestry <- coanc_admix( admix_proportions, coanc_subpops )</pre>
# FST of admixed individuals
Fst <- fst_admix( admix_proportions, coanc_subpops )</pre>
# draw all random allele freqs and genotypes, tree version
\verb"out <- draw_all_admix( admix_proportions, tree_subpops = tree_subpops, m_loci = m_loci )"
# genotypes
X <- out$X
# ancestral allele frequencies (AFs)
p_anc <- out$p_anc</pre>
# OR... draw tree subpops (intermediate) AFs separately
p_subpops_tree <- draw_p_subpops_tree( p_anc, tree_subpops )</pre>
```

coanc\_admix

Construct the coancestry matrix of an admixture model

# **Description**

The n-by-n coancestry matrix Theta of admixed individuals is determined by the n-by-k admixture proportion matrix Q and the k-by-k intermediate subpopulation coancestry matrix Psi, given by Theta = Q %\*% Psi %\*% t(Q). In the more restricted BN-PSD model, Psi is a diagonal matrix (with FST values for the intermediate subpopulations along the diagonal, zero values off-diagonal).

# Usage

```
coanc_admix(admix_proportions, coanc_subpops)
```

# Arguments

```
admix_proportions
```

The n-by-k admixture proportion matrix

10 coanc\_to\_kinship

coanc\_subpops

The intermediate subpopulation coancestry, given either as a k-by-k matrix (for the complete admixture model), or the length-k vector of intermediate subpopulation FST values (for the BN-PSD model; implies zero coancestry between subpopulations), or a scalar FST value shared by all intermediate subpopulations (also implies zero coancestry between subpopulations).

#### **Details**

As a precaution, function stops if both inputs have names and the column names of admix\_proportions and the names in coanc\_subpops disagree, which might be because these two matrices are not aligned or there is some other inconsistency.

#### Value

The n-by-n coancestry matrix.

# **Examples**

```
# a trivial case: unadmixed individuals from independent subpopulations
# number of individuals and subpops
n_ind <- 5
# unadmixed individuals
admix_proportions <- diag(rep.int(1, n_ind))</pre>
# equal Fst for all subpops
coanc_subpops <- 0.2</pre>
# diagonal coancestry matryx
coancestry <- coanc_admix(admix_proportions, coanc_subpops)</pre>
# a more complicated admixture model
# number of individuals
n_{ind} < -5
# number of intermediate subpops
k_subpops <- 2
# non-trivial admixture proportions
admix_proportions <- admix_prop_1d_linear(n_ind, k_subpops, sigma = 1)</pre>
# different Fst for each of the k_subpops
coanc_subpops <- c(0.1, 0.3)
# non-trivial coancestry matrix
coancestry <- coanc_admix(admix_proportions, coanc_subpops)</pre>
```

coanc\_to\_kinship

Transform coancestry matrix to kinship matrix

# Description

If Theta is the coancestry matrix and Phi is the kinship matrix (both are n-by-n symmetric), then these matrices agree off-diagonal, but the diagonal gets transformed as diag(Phi) = (1 + diag(Theta)) / 2.

coanc\_tree 11

# Usage

```
coanc_to_kinship(coancestry)
```

# **Arguments**

coancestry The n-by-n coancestry matrix

#### Value

The n-by-n kinship matrix, preserving column and row names.

# See Also

The inverse function is given by popkin::inbr\_diag().

## **Examples**

```
# a trivial case: unadmixed individuals from independent subpopulations
# number of individuals/subpops
n_{ind} < -5
# unadmixed individuals
admix_proportions <- diag(rep.int(1, n_ind))</pre>
# equal Fst for all subpops
inbr_subpops <- 0.2</pre>
# diagonal coancestry matryx
coancestry <- coanc_admix(admix_proportions, inbr_subpops)</pre>
kinship <- coanc_to_kinship(coancestry)</pre>
# a more complicated admixture model
# number of individuals
n_{ind} < -5
# number of intermediate subpops
k_subpops <- 2
# non-trivial admixture proportions
admix_proportions <- admix_prop_1d_linear(n_ind, k_subpops, sigma = 1)</pre>
# different Fst for each of the k subpops
inbr_subpops <- c(0.1, 0.3)
# non-trivial coancestry matrix
coancestry <- coanc_admix(admix_proportions, inbr_subpops)</pre>
kinship <- coanc_to_kinship( coancestry )</pre>
```

coanc\_tree

Calculate coancestry matrix corresponding to a tree

## **Description**

This function calculates the coancestry matrix of the subpopulations which are tip nodes in the input tree. The edges of this tree are coancestry values relative to the parent node of each child node.

12 coanc\_tree

# Usage

```
coanc_tree(tree)
```

## **Arguments**

tree

The coancestry tree relating the k\_subpops subpopulations. Must be a phylo object from the ape package (see ape::read.tree()). This tree may have a valid root edge (non-NULL tree\$root.edge between 0 and 1), which is incorporated in the output calculations.

# **Details**

The calculation takes into account that total coancestries are non-linear functions of the per-edge coancestries. Interestingly, the calculation can be simplified by a simple transformation performed by tree\_additive(), see that for more information. The self-coancestry (diagonal values) are the total coancestries of the tip nodes. The coancestry between different subpopulations is the total coancestry of their last common ancestor node.

# Value

The k\_subpops-by-k\_subpops coancestry matrix. The order of subpopulations along the rows and columns of this matrix matches tree\$tip.label. The tip labels of the tree are copied to the row and column names of this matrix.

#### See Also

fit\_tree() for the inverse function (when applied to coancestry matrices generated by a tree without noise).

tree\_additive() for calculating the additive edges. This function is called internally by coanc\_tree but the additive edges are not returned here, so call tree\_additive() if you desired them.

```
# for simulating a tree with `rtree`
library(ape)

# a typical, non-trivial example
# number of tip subpopulations
k_subpops <- 3
# simulate a random tree
tree_subpops <- rtree( k_subpops )
# coancestry matrix of subpopulations
coancestry <- coanc_tree( tree_subpops )</pre>
```

draw\_all\_admix 13

draw_all_admix	Simulate random allele frequencies and genotypes from the BN-PSD admixture model

# Description

This function returns simulated ancestral, intermediate, and individual-specific allele frequencies and genotypes given the admixture structure, as determined by the admixture proportions and the vector or tree of intermediate subpopulation FST values. The function is a wrapper around draw\_p\_anc(), draw\_p\_subpops()/draw\_p\_subpops\_tree(), make\_p\_ind\_admix(), and draw\_genotypes\_admix() with additional features such as requiring polymorphic loci. Importantly, by default fixed loci (where all individuals were homozygous for the same allele) are re-drawn from the start (starting from the ancestral allele frequencies) so no fixed loci are in the output and no biases are introduced by re-drawing genotypes conditional on any of the previous allele frequencies (ancestral, intermediate, or individual-specific). Below m\_loci (also m) is the number of loci, n is the number of individuals, and k is the number of intermediate subpopulations.

## Usage

```
draw_all_admix(
  admix_proportions,
  inbr_subpops = NULL,
 m_loci,
  tree_subpops = NULL,
 want_genotypes = TRUE,
 want_p_ind = FALSE,
 want_p_subpops = FALSE,
 want_p_anc = TRUE,
  verbose = FALSE,
  require_polymorphic_loci = TRUE,
  maf_min = 0,
 beta = NA,
  p_anc = NULL,
  p_anc_distr = NULL
)
```

## **Arguments**

admix\_proportions

The n-by-k matrix of admixture proportions.

inbr\_subpops The length-k vector (or scalar) of intermediate subpopulation FST values. Either

this or tree\_subpops must be provided (but not both).

m\_loci The number of loci to draw.

tree\_subpops The coancestry tree relating the k intermediate subpopulations. Must be a phylo

object from the ape package (see ape::read.tree()). Either this or inbr\_subpops

must be provided (but not both).

14 draw\_all\_admix

want\_genotypes If TRUE (default), includes the matrix of random genotypes in the return list.

If TRUE (NOT default), includes the matrix of individual-specific allele frequenwant\_p\_ind

cies in the return list. Note that by default p\_ind is not constructed in full at all, instead a fast low-memory algorithm constructs it in parts as needed only; beware that setting want\_p\_ind = TRUE increases memory usage in comparison.

If TRUE (NOT default), includes the matrix of random intermediate subpopulawant\_p\_subpops

tion allele frequencies in the return list.

If TRUE (default), includes the vector of random ancestral allele frequencies in want\_p\_anc

the return list.

verbose If TRUE, prints messages for every stage in the algorithm.

require\_polymorphic\_loci

If TRUE (default), returned genotype matrix will not include any fixed loci (loci that happened to be fixed are drawn again, starting from their ancestral allele frequencies, and checked iteratively until no fixed loci remain, so that the final

number of polymorphic loci is exactly m\_loci).

maf\_min The minimum minor allele frequency (default zero), to extend the working defi-

> nition of "fixed" above to include rare variants. This helps simulate a frequencybased locus ascertainment bias. Loci with minor allele frequencies less than or equal to this value are treated as fixed (passed to fixed\_loci()). This parame-

ter has no effect if require\_polymorphic\_loci is FALSE.

beta Shape parameter for a symmetric Beta for ancestral allele frequencies p\_anc. If

> NA (default), p\_anc is uniform with range in [0.01, 0.5]. Otherwise, p\_anc has a symmetric Beta distribution with range in [0, 1]. Has no effect if either p\_anc

or p\_anc\_distr options are non-NULL.

If provided, it is used as the ancestral allele frequencies (instead of drawing p\_anc

> random ones). Must either be a scalar or a length-m\_loci vector. If scalar and want\_p\_anc = TRUE, then the returned p\_anc is the scalar value repeated m\_loci times (it is always a vector). If a locus was fixed and has to be redrawn, the ancestral allele frequency in p\_anc is retained and only downstream allele

frequencies and genotypes are redrawn (contrast to p\_anc\_distr below).

If provided, ancestral allele frequencies are drawn with replacement from this p\_anc\_distr

> vector (which may have any length) or function, instead of from draw\_p\_anc(). If a function, must accept a single parameter specifying the number of loci to draw. If a locus was fixed and has to be redrawn, the ancestral allele frequency

is redrawn from the distribution (contrast to p\_anc above).

#### **Details**

As a precaution, function stops if both the column names of admix\_proportions and the names in inbr\_subpops or tree\_subpops exist and disagree, which might be because these two data are not aligned or there is some other inconsistency.

#### Value

A named list with the following items (which may be missing depending on options):

• X: An m-by-n matrix of genotypes. Included if want\_genotypes = TRUE.

draw\_genotypes\_admix

- p\_anc: A length-m vector of ancestral allele frequencies. Included if want\_p\_anc = TRUE.
- p\_subpops: An m-by-k matrix of intermediate subpopulation allele frequencies Included if want\_p\_subpops = TRUE.
- p\_ind: An m-by-n matrix of individual-specific allele frequencies. Included if want\_p\_ind = TRUE.

```
# dimensions
# number of loci
m_loci <- 10
# number of individuals
n_ind <- 5
# number of intermediate subpops
k_subpops <- 2
# define population structure
\# FST values for k = 2 subpopulations
inbr_subpops <- c(0.1, 0.3)
# admixture proportions from 1D geography
admix_proportions <- admix_prop_1d_linear(n_ind, k_subpops, sigma = 1)</pre>
# draw all random allele freqs and genotypes
out <- draw_all_admix(admix_proportions, inbr_subpops, m_loci)</pre>
# return value is a list with these items:
# genotypes
X \leftarrow out$X
# ancestral AFs
p_anc <- out$p_anc</pre>
# # these are excluded by default, but would be included if ...
# # ... `want_p_subpops == TRUE`
# # intermediate subpopulation AFs
# p_subpops <- out$p_subpops</pre>
# # ... `want_p_ind == TRUE`
# # individual-specific AFs
# p_ind <- out$p_ind</pre>
```

## **Description**

Given the Individual-specific Allele Frequency (IAF) matrix p\_ind for m loci (rows) and n individuals (columns), the genotype matrix X (same dimensions as p\_ind) is drawn from the Binomial distribution equivalent to X[i, j] <- rbinom(1, 2, p\_ind[i, j]), except the function is more efficient. If admix\_proportions is provided as the second argument (a matrix with n individuals along rows and k intermediate subpopulations along the columns), the first argument p\_ind is treated as the intermediate subpopulation allele frequency matrix (must be m-by-k) and the IAF matrix is equivalent to p\_ind %\*% t(admix\_proportions). However, in this case the function computes the IAF matrix in parts only, never stored in full, greatly reducing memory usage. If admix\_proportions is missing, then p\_ind is treated as the IAF matrix.

# Usage

```
draw_genotypes_admix(p_ind, admix_proportions = NULL)
```

# **Arguments**

p\_ind

The m-by-n IAF matrix (if admix\_proportions is missing) or the m-by-k intermediate subpopulation allele frequency matrix (if admix\_proportions is present).

admix\_proportions

The optional n-by-k admixture proportion matrix (to draw data from the admixture model using reduced memory, by not fully forming the IAF matrix). If provided, and if both admix\_proportions and p\_ind have column names, and if they disagree, the function stops as a precaution, as this suggests the data is misaligned or inconsistent in some way.

# Value

The m-by-n genotype matrix. If admix\_proportions is missing, the row and column names of p\_ind are copied to this output. If admix\_proportions is present, the row names of the output are the row names of p\_ind, while the column names of the output are the row names of admix\_proportions.

```
# dimensions
# number of loci
m_loci <- 10
# number of individuals
n_ind <- 5
# number of intermediate subpops
k_subpops <- 2

# define population structure
# FST values for k = 2 subpops
inbr_subpops <- c(0.1, 0.3)
# non-trivial admixture proportions
admix_proportions <- admix_prop_1d_linear(n_ind, k_subpops, sigma = 1)</pre>
```

draw\_p\_anc 17

```
# draw allele frequencies
# vector of ancestral allele frequencies
p_anc <- draw_p_anc(m_loci)

# matrix of intermediate subpop allele freqs
p_subpops <- draw_p_subpops(p_anc, inbr_subpops)

# matrix of individual-specific allele frequencies
p_ind <- make_p_ind_admix(p_subpops, admix_proportions)

# draw genotypes from intermediate subpops (one individual each)
X_subpops <- draw_genotypes_admix(p_subpops)

# and genotypes for admixed individuals
X_ind <- draw_genotypes_admix(p_ind)

# draw genotypes for admixed individuals without p_ind intermediate
# (p_ind is computed internally in parts, never stored in full,
# reducing memory use substantially)
X_ind <- draw_genotypes_admix(p_subpops, admix_proportions)</pre>
```

draw\_p\_anc

Draw random Uniform or Beta ancestral allele frequencies

# **Description**

This is simply a wrapper around stats::runif() or stats::rbeta() (depending on parameters) with different defaults and additional validations.

# Usage

```
draw_p_anc(m_loci, p_min = 0.01, p_max = 0.5, beta = NA)
```

# Arguments

m_loci	Number of loci to draw.
p_min	Minimum allele frequency to draw (Uniform case only).
p_max	Maximum allele frequency to draw (Uniform case only).
beta	Shape parameter for a symmetric Beta. If NA (default), Uniform(p_min, p_max) is used. Otherwise, a Symmetric Beta is used and the user-specified range is ignored (values in [0, 1] will be returned).

#### Value

A length-m vector of random ancestral allele frequencies

18 draw\_p\_subpops

# **Examples**

```
# Default is uniform with range between 0.01 and 0.5
p_anc <- draw_p_anc(m_loci = 10)

# Use of `beta` triggers a symmetric Beta distribution.
# This parameter has increased density for rare minor allele frequencies,
# resembling the 1000 Genomes allele frequency distribution
p_anc <- draw_p_anc(m_loci = 10, beta = 0.03)</pre>
```

draw\_p\_subpops

Draw allele frequencies for independent subpopulations

# **Description**

The allele frequency matrix P for m\_loci loci (rows) and k\_subpops independent subpopulations (columns) are drawn from the Balding-Nichols distribution with ancestral allele frequencies p\_anc and FST parameters inbr\_subpops equivalent to P[i, j] <- rbeta(1, nu\_j \* p\_anc[i], nu\_j \* (1 - p\_anc[i])), where nu\_j <- 1 / inbr\_subpops[j] - 1. The actual function is more efficient than the above code.

## **Usage**

```
draw_p_subpops(p_anc, inbr_subpops, m_loci = NA, k_subpops = NA)
```

#### **Arguments**

p\_anc The scalar or length-m\_loci vector of ancestral allele frequencies per locus.

inbr\_subpops The scalar or length-k\_subpops vector of subpopulation FST values.

m\_loci If p\_anc is scalar, optionally provide the desired number of loci (lest only one

locus be simulated). Stops if both  $length(p_anc) > 1$  and  $m_loci$  is not NA and

they disagree.

k\_subpops If inbr\_subpops is a scalar, optionally provide the desired number of subpopu-

lations (lest a single subpopulation be simulated). Stops if both length(inbr\_subpops)

> 1 and k\_subpops is not NA and they disagree.

#### Value

The m\_loci-by-k\_subpops matrix of independent subpopulation allele frequencies. If p\_anc is length-m\_loci with names, these are copied to the row names of this output matrix. If inbr\_subpops is length-k\_subpops with names, these are copied to the column names of this output matrix.

#### See Also

draw\_p\_subpops\_tree() for version for subpopulations related by a tree, which can therefore be non-independent.

draw\_p\_subpops\_tree 19

# **Examples**

```
# a typical, non-trivial example
# number of loci
m_loci <- 10
# random vector of ancestral allele frequencies
p_anc <- draw_p_anc(m_loci)</pre>
# FST values for two subpops
inbr_subpops <- c(0.1, 0.3)
# matrix of intermediate subpop allele freqs
p_subpops <- draw_p_subpops(p_anc, inbr_subpops)</pre>
# special case of scalar p_anc
p_subpops <- draw_p_subpops(p_anc = 0.5, inbr_subpops, m_loci = m_loci)</pre>
stopifnot ( nrow( p_subpops ) == m_loci )
# special case of scalar inbr_subpops
k_subpops <- 2
p_subpops <- draw_p_subpops(p_anc, inbr_subpops = 0.2, k_subpops = k_subpops)</pre>
stopifnot ( ncol( p_subpops ) == k_subpops )
# both main parameters scalars but return value still matrix
p_subpops <- draw_p_subpops(p_anc = 0.5, inbr_subpops = 0.2, m_loci = m_loci, k_subpops = k_subpops)
stopifnot ( nrow( p_subpops ) == m_loci )
stopifnot ( ncol( p_subpops ) == k_subpops )
# passing scalar parameters without setting dimensions separately results in a 1x1 matrix
p_subpops <- draw_p_subpops(p_anc = 0.5, inbr_subpops = 0.2)</pre>
stopifnot ( nrow( p_subpops ) == 1 )
stopifnot ( ncol( p_subpops ) == 1 )
```

# **Description**

The allele frequency matrix P for m\_loci loci (rows) and k\_subpops subpopulations (columns) are drawn from the Balding-Nichols distribution. If the allele frequency in the parent node is p and the FST parameter of the child node from the parent node is F, then the allele frequency in the child node is drawn from rbeta(1, nu \* p, nu \* (1 - p)), where nu < - 1 / F - 1. This function iterates drawing allele frequencies through the tree structure, therefore allowing covariance between subpopulations that share branches.

#### Usage

```
draw_p_subpops_tree(p_anc, tree_subpops, m_loci = NA, nodes = FALSE)
```

## **Arguments**

p\_anc The scalar or length-m\_loci vector of ancestral allele frequencies per locus.

tree\_subpops The coancestry tree relating the k\_subpops subpopulations. Must be a phylo

object from the ape package (see ape::read.tree()). The edge lengths of this tree must be the FST parameters relating parent and child subpopulations.

m\_loci If p\_anc is scalar, optionally provide the desired number of loci (lest only one

locus be simulated). Stops if both length( $p_anc$ ) > 1 and  $m_loci$  is not NA and

they disagree.

nodes If FALSE (default), returns allele frequencies of "tip" subpopulations only; oth-

erwise returns all allele frequencies, including internal nodes.

#### Value

The m\_loci-by-k\_subpops matrix of independent subpopulation allele frequencies. If nodes = FALSE, columns include only tip subpopulations. If nodes = TRUE, internal node subpopulations are also included (in this case the input p\_anc is returned in the column corresponding to the root node). In all cases subpopulations are ordered as indexes in the tree, which normally implies the tip nodes are listed first, followed by the internal nodes (as in tree\_subpops\$edge matrix, see ape::read.tree() for details). The tree\_subpops tip and node names are copied to the column names of this output matrix (individual names may be blank if missing in tree (such as for internal nodes); column names are NULL if all tree\_subpops tip labels were blank, regardless of internal node labels). If p\_anc is length-m\_loci with names, these names are copied to the row names of this output matrix.

## See Also

```
draw_p_subpops() for version for independent subpopulations.
```

```
For "phylo" tree class, see ape::read.tree()
```

```
# for simulating a tree with `rtree`
library(ape)

# a typical, non-trivial example
# number of tip subpopulations
k_subpops <- 3
# number of loci
m_loci <- 10
# random vector of ancestral allele frequencies
p_anc <- draw_p_anc(m_loci)
# simulate tree
tree_subpops <- rtree( k_subpops )
# matrix of intermediate subpop allele freqs
p_subpops <- draw_p_subpops_tree(p_anc, tree_subpops)</pre>
```

fit\_tree 21

fit\_tree

Fit a tree structure to a coancestry matrix

## **Description**

Implements a heuristic algorithm to find the optimal tree topology based on joining pairs of sub-populations with the highest between-coancestry values, and averaging parent coancestries for the merged nodes (a variant of WPGMA hierarchical clustering stats::hclust()). Branch lengths are optimized using the non-negative least squares approach nnls::nnls(), which minimize the residual square error between the given coancestry matrix and the model coancestry implied by the tree. This algorithm recovers the true tree when the input coancestry truly belongs to this tree and there is no estimation error (i.e. it is the inverse function of coanc\_tree()), and performs well for coancestry estimates (i.e. the result of simulating genotypes from the true tree, i.e. from draw\_all\_admix(), and estimating the coancestry using popkin::popkin() followed by popkin::inbr\_diag()).

# Usage

```
fit_tree(coancestry, method = "mcquitty")
```

# **Arguments**

coancestry The coancestry matrix to fit a tree to.

method The hierarchical clustering method (passed to stats::hclust()). While all

stats::hclust() methods are supported, only two really make sense for this application: "mcquitty" (i.e. WPGMA, default) and "average" (UPGMA).

# **Details**

The tree is bifurcating by construction, but edges may equal zero if needed, effectively resulting in multifurcation, although this code makes no attempt to merge nodes with zero edges. For best fit to arbitrary data, the root edge is always fit to the data (may also be zero). Data fit may be poor if the coancestry does not correspond to a tree, particularly if there is admixture between subpopulations.

### Value

A phylo object from the ape package (see ape::read.tree()), with two additional list elements at the end:

- edge: (standard phylo.) A matrix describing the tree topology, listing parent and child node indexes.
- Nnode: (standard phylo.) Number of internal (non-leaf) nodes.
- tip.label: (standard phylo.) Labels for tips (leaf nodes), in order of index as in edge matrix above. These match the row names of the input coancestry matrix, or if names are missing, the row indexes of this matrix are used (in both cases, labels may be reordered compared to rownames (coancestry)). Tips are ordered as they appear in the above edge matrix (ensuring visual agreement in plots between the tree and its resulting coancestry matrix via

22 fixed\_loci

coanc\_tree()), and in an order that matches the input coancestry's subpopulation order as
much as possible (tree constraints do not permit arbitrary tip orders, but a heuristic implemented in tree\_reorder() is used to determine a reasonable order when an exact match is
not possible).

- edge.length: (standard phylo.) Values of edge lengths in same order as rows of edge matrix above.
- root.edge: (standard phylo.) Value of root edge length.
- rss: The residual sum of squares of the model coancestry versus the input coancestry.
- edge.length.add: Additive edge values (regarding their effect on coancestry, as opposed to edge.length which are probabilities, see coanc\_tree())

# See Also

```
coanc_tree()) for the inverse function (when the coancestry corresponds exactly to a tree).

tree_reorder() for reordering tree structure so that tips appear in a particular desired order.
```

# **Examples**

```
# create a random tree
library(ape)
k <- 5
tree <- rtree( k )
# true coancestry matrix for this tree
coanc <- coanc_tree( tree )

# now recover tree from this coancestry matrix:
tree2 <- fit_tree( coanc )
# tree2 equals tree!</pre>
```

fixed\_loci

Identify fixed loci

# Description

A locus is "fixed" if the non-missing sub-vector contains all 0's or all 2's (the locus is completely homozygous for one allele or completely homozygous for the other allele). This function tests each locus, returning a vector that is TRUE for each fixed locus, FALSE otherwise. Loci with only missing elements (NA) are treated as fixed. The parameter maf\_min extends the "fixed" definition to loci whose minor allele frequency is smaller or equal than this value. Below m is the number of loci, and n is the number of individuals.

# Usage

```
fixed_loci(X, maf_min = 0)
```

fst\_admix 23

#### **Arguments**

Χ

The m-by-n genotype matrix

maf\_min

The minimum minor allele frequency (default zero), to extend the working definition of "fixed" to include rare variants. Loci with minor allele frequencies less than or *equal* to this value are marked as fixed. Must be a scalar between 0 and 0.5.

#### Value

A length-m boolean vector where the i element is TRUE if locus i is fixed or completely missing, FALSE otherwise. If X had row names, they are copied to the names of this output vector.

# **Examples**

```
# here's a toy genotype matrix
X <- matrix(</pre>
       data = c(
              2, 2, NA, # fixed locus (with one missing element)
              0, NA, 0, # another fixed locus, for opposite allele
                        # NOT fixed (heterozygotes are not considered fixed)
              1, 1, 1,
              0, 1, 2, # a completely variable locus
              0, 0, 1, # a somewhat "rare" variant
              NA, NA, NA # completely missing locus (will be treated as fixed)
             ),
       ncol = 3, byrow = TRUE)
# test that we get the desired values
stopifnot(
  fixed_loci(X) == c(TRUE, TRUE, FALSE, FALSE, FALSE, TRUE)
# the "rare" variant gets marked as "fixed" if we set `maf_min` to its frequency
  fixed_loci(X, maf_min = 1/6) == c(TRUE, TRUE, FALSE, FALSE, TRUE, TRUE)
```

fst\_admix

Calculate FST for the admixed individuals

# **Description**

This function returns the generalized FST of the admixed individuals given their admixture proportion matrix, the coancestry matrix of intermediate subpopulations (or its special cases, see coanc\_subpops parameter below), and optional weights for individuals. This FST equals the weighted mean of the diagonal of the coancestry matrix (see coanc\_admix()). Below there are n individuals and k intermediate subpopulations.

24 fst\_admix

# Usage

```
fst_admix(admix_proportions, coanc_subpops, weights = NULL)
```

# **Arguments**

admix\_proportions

The n-by-k admixture proportion matrix

coanc\_subpops

Either the k-by-k intermediate subpopulation coancestry matrix (for the complete admixture model), or the length-k vector of intermediate subpopulation FST values (for the BN-PSD model; assumes zero coancestries between subpopulations), or a scalar FST value shared by all intermediate subpopulations

(also assumes zero coancestry between subpopulations).

weights

Optional length-n vector of weights for individuals that define their generalized

FST (default uniform weights)

#### **Details**

As a precaution, function stops if both inputs have names and the column names of admix\_proportions and the names in coanc\_subpops disagree, which might be because these two matrices are not aligned or there is some other inconsistency.

#### Value

The generalized FST of the admixed individuals

```
# set desired parameters
# number of individuals
n ind <- 1000
# number of intermediate subpopulations
k_subpops <- 10
# differentiation of intermediate subpopulations
coanc_subpops <- ( 1 : k_subpops ) / k_subpops</pre>
# construct admixture proportions
admix_proportions <- admix_prop_1d_linear(n_ind, k_subpops, sigma = 1)</pre>
# lastly, calculate Fst!!! (uniform weights in this case)
fst_admix(admix_proportions, coanc_subpops)
```

make\_p\_ind\_admix 25

make_p_ind_admix	Construct individual-specific allele frequency matrix under the PSD admixture model
	aamixiure moaei

# **Description**

Here m is the number of loci, n the number of individuals, and k the number of intermediate subpopulations. The m-by-n individual-specific allele frequency matrix p\_ind is constructed from the m-by-k intermediate subpopulation allele frequency matrix p\_subpops and the n-by-k admixture proportion matrix admix\_proportions equivalent to p\_ind <- p\_subpops %\*% t(admix\_proportions). This function is a wrapper around tcrossprod(), but also ensures the output allele frequencies are in [0, 1] (otherwise not guaranteed due to limited machine precision).

# Usage

```
make_p_ind_admix(p_subpops, admix_proportions)
```

## **Arguments**

```
p_subpops The m-by-k matrix of intermediate subpopulation allele frequencies. admix_proportions
```

The n-by-k matrix of admixture proportions.

# **Details**

If both admix\_proportions and p\_subpops have column names, and if they disagree, the function stops as a precaution, as this suggests the data is misaligned or inconsistent in some way.

#### Value

The m-by-n matrix of individual-specific allele frequencies p\_ind. Row names equal those from p\_subpops, and column names equal rownames from admix\_proportions, if present.

```
# data dimensions
# number of loci
m_loci <- 10
# number of individuals
n_ind <- 5
# number of intermediate subpops
k_subpops <- 2
# FST values for k = 2 subpops
inbr_subpops <- c(0.1, 0.3)
# non-trivial admixture proportions
admix_proportions <- admix_prop_1d_linear(n_ind, k_subpops, sigma = 1)</pre>
```

26 scale\_tree

```
# random vector of ancestral allele frequencies
p_anc <- draw_p_anc(m_loci)

# matrix of intermediate subpop allele freqs
p_subpops <- draw_p_subpops(p_anc, inbr_subpops)

# matrix of individual-specific allele frequencies
p_ind <- make_p_ind_admix(p_subpops, admix_proportions)</pre>
```

scale\_tree

Scale a coancestry tree

# **Description**

Scale by a scalar factor all the edges (\$edge.length) of a phylo object from the ape package, including the root edge (\$root.edge) if present, and additive edges (\$edge.length.add, present in trees returned by fit\_tree()). Stops if any of the edges exceed 1 before or after scaling (since these edges are IBD probabilities).

## Usage

```
scale_tree(tree, factor)
```

# **Arguments**

tree The coancestry tree to edit.

factor The scalar factor to multiply all edges. Must be non-negative, and not be so

large that any edge exceeds 1 after scaling.

# Value

The edited tree with all edges scaled as desired.

## See Also

```
ape::read.tree() for reading phylo objects and their definition.
```

```
# create a random tree
library(ape)
k <- 5
tree <- rtree( k )

# scale this tree
tree_scaled <- scale_tree( tree, 0.5 )</pre>
```

tree\_additive 27

tree_additive	Calculate additive edges for a coancestry tree, or viceversa
ti cc_addrtive	caretiare additive edges for a conficestry free, or viceversa

# **Description**

A coancestry tree has IBD probabilities as edge values, which describe how each child and parent subpopulation in the tree is related. This means that each parameter is relative to its parent subpopulation (varies per edge), and they are not in general IBD probabilities from the root. This function computes "additive" edges that corresponds more closely with the coancestry matrix of this tree, which gives parameters relative to the root (ancestral) population (see details below). The additive edges are computed on a new element of the tree phylo object, so they do not overwrite the probabilistic edges. The reverse option assumes that the main edges of the phylo object are additive edges, then calculates the probabilistic edges and stores as the main edges and moves the original additive edges on the new element.

# Usage

```
tree_additive(tree, rev = FALSE, force = FALSE)
```

## Arguments

tree	The coancestry tree with either probabilistic edges (if rev = FALSE) or additive edges (if rev = TRUE) as the main edges (stored in tree\$edge.length). Must be a phylo object from the ape package (see ape::read.tree()). This tree may have a valid root edge (non-NULL tree\$root.edge between 0 and 1), which is incorporated in the output calculations. Function stops if the input data is not valid. Probabilistic edges are valid if and only if they are all between zero and one. Additive edges are valid if and only if they are all non-negative and the sum of edges between the root and each tip (leaf node) does not exceed 1.
rev	If FALSE (default), assumes the main edges are probabilistic values, and calculates additive values. If TRUE, assumes main edges are additive values, and calculates probabilistic values.
force	If FALSE (default), function stops if input tree already has additive edges (if tree\$edge.length.add is not NULL). If TRUE, these values are ignored and overwritten.

# Details

The calculation takes into account that total coancestries are non-linear combinations of the peredge coancestries. For example, if the root node is A, and subpopulation C is connected to A only through an internal node B, then its total self-coancestry coanc\_A\_C relative to A is given by coanc\_A\_B (the coancestry between A and B) and coanc\_B\_C (the coancestry between B and C) by coanc\_A\_C = coanc\_A\_B + coanc\_B\_C \* (1 - coanc\_A\_B). This transformation ensures that the total coancestry is a probability that does not exceed one even when the per-edge coancestries sum to a value greater than one. The "additive" edge for B and C is coanc\_B\_C \* (1 - coanc\_A\_B), so it is the probabilistic edge coanc\_B\_C shrunk by 1 - coanc\_A\_B, which can then just be added to the parent edge coanc\_A\_B to give the total coancestry coanc\_A\_C. This transformation is iterated

28 tree\_additive

for all nodes in the tree, noting that roots B connected to the root node A have equal probabilistic and additive edges coanc\_A\_B (unless the tree has a root edge, in which case that one is used to transform as above), and the edge of a node C not directly connected to a root uses the calculated edge coanc\_A\_C as above to shrink its children's edges into the additive scale.

#### Value

The input phylo object extended so that the main edges (tree\$edge.length) are probabilistic edges, and the additive edges are stored in tree\$edge.length.add. This is so for both values of rev

#### See Also

coanc\_tree(), the key application facilitated by additive edges.

```
# for simulating a tree with `rtree`
library(ape)
# SETUP: number of tip subpopulations
k <- 5
# simulate a random tree
\# edges are drawn from Uniform(0, 1), so these are valid probabilistic edges
tree <- rtree( k )</pre>
# inspect edges
tree$edge.length
# RUN calculate additive edges (safe to overwrite object)
tree <- tree_additive( tree )</pre>
# inspect edges again
# probabilistic edges are still main edges:
tree$edge.length
# additive edges are here
tree$edge.length.add
# let's go backwards now, starting from the additive edges
# SETUP
# these are harder to simulate, so let's copy the previous value to the main edges
tree$edge.length <- tree$edge.length.add</pre>
# and delete the extra entry (if it's present, function stops)
tree$edge.length.add <- NULL</pre>
# inspect before
tree$edge.length
# RUN reverse version (safe to overwrite object again)
tree <- tree_additive( tree, rev = TRUE )</pre>
# inspect after
# probabilistic edges are main edges:
tree$edge.length
# additive edges (previously main edges) were moved here
```

tree\_reindex\_tips 29

tree\$edge.length.add

tree\_reindex\_tips

Reindex tree tips in order of appearance in edges

# **Description**

The phylo object from the ape package (see ape::read.tree()) permits mismatches between the order of tips as present in the tip labels vector (tree\$tip.label) and in the edge matrix (tree\$edge), which can cause mismatches in plots and other settings. This function reindexes edges and tips so that they agree in tip order.

# Usage

tree\_reindex\_tips(tree)

#### **Arguments**

tree

A phylo object from the ape package (see ape::read.tree()). Works with standard phylo objects, and also with our extended trees (in that additive edges tree\$edge.length.add are recalculated after reordering if they were present).

# **Details**

Order mismatches between tip labels vector and edge matrix can lead to disagreeing order downstream, in variables that order tips as in the labels vector (such as the coancestry matrices output by our coanc\_tree()) and tree plots (whose order is guided by the edge matrix, particularly when the edge matrix is ordered by clade as in ape::reorder.phylo()).

This function first reorders the edges of the input tree using ape::reorder.phylo() with default option order = 'cladewise', which will list edges and tip nodes in plotting order. Then tips are indexed so that the first tip to appear has index 1 in the edge matrix (and consequently appears first in the tip labels vector), the second has index 2, and so on. Thus, the output tree has both edges and tips reordered, to have a consistent tip order and best experience visualizing trees and their coancestry matrices.

# Value

The modified tree (phylo object) with reordered edges and tips.

#### See Also

tree\_reorder() for reordering tree structure so that tips appear in a particular desired order.

30 tree\_reorder

# **Examples**

```
# create a random tree
library(ape)
k <- 5
tree <- rtree( k )</pre>
# trees constructed this way are already ordered as desired, so this function doesn't change it:
tree2 <- tree_reindex_tips( tree )</pre>
# tree2 equals tree!
# let's scramble the edges on purpose
# to create an example where reindexing is needed
tree_rand <- tree</pre>
# new order of edges
indexes <- sample( Nedge( tree_rand ) )</pre>
# reorder all edge values
tree_rand$edge <- tree_rand$edge[ indexes, ]</pre>
tree_rand$edge.length <- tree_rand$edge.length[ indexes ]</pre>
# now let's reorder edges slightly so tree is more reasonable-looking
# (otherwise plot looks tangled)
tree_rand <- reorder( tree_rand, order = 'postorder' )</pre>
# you can now see that, unless permutation got lucky,
# the order of the tip labels in the vector and on the plot disagree:
tree_rand$tip.label
plot( tree_rand )
# now reorder tips to match plotting order (as defined in the `edge` matrix)
tree_rand <- tree_reindex_tips( tree_rand )</pre>
# now tip labels vector and plot should agree in order:
# (plot was not changed)
tree_rand$tip.label
plot( tree_rand )
```

tree\_reorder

Reorder tree tips to best match a desired order

# Description

This functions reorganizes the tree structure so that its tips appear in a desired order if possible, or in a reasonably close order when an exact solution is impossible. This tip order in the output tree is the same in both the tip labels vector (tree\$tip.label) and edge matrix (tree\$edge), ensured by using tree\_reindex\_tips() internally.

# Usage

```
tree_reorder(tree, labels)
```

tree\_reorder 31

# **Arguments**

tree A phylo object from the ape package (see ape::read.tree()). Works with

standard phylo objects, and also with our extended trees (in that additive edges tree\$edge.length.add are recalculated after reordering if they were present).

labels A character vector with all tip labels in the desired order. Must contain each tip

label in tree exactly once.

#### **Details**

This function has the same goal as ape::rotateConstr(), which implements a different heuristic algorithm that did not perform well in our experience.

# Value

The modified tree (phylo object) with reordered edges and tips.

#### See Also

tree\_reindex\_tips() to reorder tips in the labels vector to match the edge matrix order, which ensures agreement in plots (assuming plot show desired order already).

```
# create a random tree
library(ape)
k <- 5
tree <- rtree( k )</pre>
# let's set the current labels as the desired order
labels <- tree$tip.label</pre>
# now let's scramble the edges on purpose
# to create an example where reordering is needed
tree_rand <- tree
# new order of edges
indexes <- sample( Nedge( tree_rand ) )</pre>
# reorder all edge values
tree_rand$edge <- tree_rand$edge[ indexes, ]</pre>
tree_rand$edge.length <- tree_rand$edge.length[ indexes ]</pre>
# now let's reorder edges slightly so tree is more reasonable-looking
# (otherwise plot looks tangled)
tree_rand <- reorder( tree_rand, order = 'postorder' )</pre>
# the order of the tip labels in the vector and on the plot disagree with each other:
tree_rand$tip.label
plot( tree_rand )
# now reorder tree object so tips are in the desired order:
tree_rand <- tree_reorder( tree_rand, labels )</pre>
# now tip labels vector and plot should agree in order:
# (the original tree was recovered!)
tree_rand$tip.label
```

32 undiff\_af

```
plot( tree_rand )

# order the tree in a different way than the original order
labels <- paste0( 't', 1 : k )

# in this case, it's often impossible to get a perfect output order

# (because the tree structure constrains the possible plot orders),

# but this function tries its best to get close to the desired order
tree2 <- tree_reorder( tree, labels )
plot(tree2)</pre>
```

undiff\_af

Undifferentiate an allele distribution

# **Description**

This function takes a vector of allele frequencies and a mean kinship value, and returns a new distribution of allele frequencies that is consistent with reversing differentiation with the given kinship, in the sense that the new distribution is more concentrated around the middle (0.5) than the input/original by an amount predicted from theory. The new distribution is created by weighing the input distribution with a random mixing distribution with a lower variance. An automatic method is provided that always selects a Beta distribution with just the right concentration to work for the given data and kinship. Explicit methods are also provided for more control, but are more likely to result in errors when mixing variances are not small enough (see details below).

# Usage

```
undiff_af(
   p,
   kinship_mean,
   distr = c("auto", "uniform", "beta", "point"),
   alpha = 1,
   eps = 10 * .Machine$double.eps
)
```

# **Arguments**

n

A vector of observed allele frequencies.

kinship\_mean

The mean kinship value of the differentiation to reverse.

distr

Name of the mixing distribution to use.

- "auto" picks a symmetric Beta distribution with parameters that ensure a small enough variance to succeed.
- "beta" is a symmetric Beta distribution with parameter alpha as provided below.
- "uniform" is a uniform distribution (same as "beta" with alpha = 1).
- "point" is a distribution fully concentrated/fixed at 0.5 (same as the limit of "beta" with alpha = Inf, which has zero variance).

undiff af 33

alpha Shape parameter for distr = "beta", ignored otherwise.

eps If distr = "auto", this small value is added to the calculated alpha to avoid

roundoff errors and ensuring that the mixing variance is smaller than the maxi-

mum allowed.

#### **Details**

Model: Suppose we started from an allele frequency p0 with expectation 0.5 and variance V0. Differentiation creates a new (sample) allele frequency p1 without changing its mean (E(p1|p0) = p0) and with a conditional variance given by the mean kinship phi: Var(p1|p0) = p0\*(1-p0)\*phi. The total variance of the new distribution (calculated using the law of total variance) equals V1 = Var(p1) = phi/4 + (1-phi)\*V0. (Also E(p1) = 0.5). So the new variance is greater for Paiv = Var(p1) = 1/4 for any distribution bounded in (0,1)). Thus, given V1 and Paiv = Var(p1) = 1/4 for any distribution with the original, lower variance of V0 = (V1-phi/4)/(1-phi). An error is thrown if V1 < Paiv = Var(p1) in input data, which is inconsistent with this assumed model.

Construction of "undifferentiated" allele frequencies:  $p_out = w*p_in + (1-w)*p_mix$ , where  $p_in$  is the input with sample variance  $V_in$  (V1 in above model) and  $p_mix$  is a random draw from the mixing distribution distr with expectation 0.5 and known variance  $V_mix$ . The output variance is  $V_out = w^2*V_in + (1-w)^2*V_mix$ , which we set to the desired  $V_out = (V_in-phi/4)/(1-phi)$  (V0 in above model) and solve for w (the largest of the two quadratic roots is used). An error is thrown if  $V_mix > V_out$  (the output variance must be larger than the mixing variance). This error is avoided by manually adjusting choice of distr and alpha (for distr = "beta"), or preferably with distr = "auto" (default), which selects a Beta distribution with alpha =  $(1/(4*V_out)-1)/2$  + eps that is guaranteed to work for any valid  $V_out$  (assuming  $V_in < phi/4$ ).

## Value

A list with the new distribution and several other informative statistics, which are named elements:

- p: A new vector of allele frequencies with the same length as input p, with the desired variance (see details) obtained by weighing the input p with new random data from distribution distr.
- w: The weight used for the input data (1-w for the mixing distribution).
- kinship\_mean\_max: The maximum mean kinship possible for undifferentiating this data (equals four times the input variance (see details), which results in zero output variance).
- V\_in: sample variance of input p, assuming its expectation is 0.5.
- V\_out: target variance of output p.
- V\_mix: variance of mixing distribution.
- alpha: the value of alpha used for symmetric Beta mixing distribution, informative if distr = "auto".

```
# create random uniform data for these many loci
m <- 100
p <- runif( m )
# differentiate the distribution using Balding-Nichols model
F <- 0.1</pre>
```

34 undiff\_af

```
nu <- 1 / F - 1
p2 <- rbeta( m, p * nu, (1 - p) * nu )

# now undifferentiate with this function
# NOTE in this particular case `F` is also the mean kinship
# default "automatic" distribution recommended
# (avoids possible errors for specific distributions)
p3 <- undiff_af( p2, F )$p

# note p3 does not equal p exactly (original is unrecoverable)
# but variances (assuming expectation is 0.5 for all) should be close to each other,
# and both be lower than p2's variance:
V1 <- mean( ( p - 0.5 )^2 )
V2 <- mean( ( p2 - 0.5 )^2 )
V3 <- mean( ( p3 - 0.5 )^2 )
# so p3 is stochastically consistent with p as far as the variance is concerned</pre>
```

# **Index**

```
admix_prop_1d_circular, 2
                                                popkin::popkin(), 21
admix_prop_1d_linear, 4
                                                scale_tree, 26
admix_prop_1d_linear(), 3
                                                stats::hclust(), 21
admix_prop_indep_subpops, 6
                                                 stats::rbeta(), 17
ape::read.tree(), 12, 13, 20, 21, 26, 27, 29,
                                                stats::runif(), 17
ape::reorder.phylo(), 29
                                                 tcrossprod(), 25
ape::rotateConstr(), 31
                                                 tree_additive, 27
                                                 tree_additive(), 12
bnpsd, 7
                                                tree_reindex_tips, 29
bnpsd-package (bnpsd), 7
                                                tree_reindex_tips(), 30, 31
coanc_admix, 9
                                                tree_reorder, 30
                                                 tree_reorder(), 22, 29
coanc_admix(), 23
coanc_to_kinship, 10
                                                undiff_af, 32
coanc_tree, 11
coanc_tree(), 21, 22, 28, 29
draw_all_admix, 13
draw_all_admix(), 21
draw_genotypes_admix, 15
draw_genotypes_admix(), 13
draw_p_anc, 17
draw_p_anc(), 13, 14
draw_p_subpops, 18
draw_p_subpops(), 13, 20
draw_p_subpops_tree, 19
draw_p_subpops_tree(), 13, 18
fit_tree, 21
fit_tree(), 12, 26
fixed_loci, 22
fixed_loci(), 14
fst_admix, 23
make_p_ind_admix, 25
make_p_ind_admix(), 13
nnls::nnls(), 21
popkin::inbr_diag(), 11, 21
```