

# Package ‘asymmetry.measures’

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**Type** Package

**Title** Asymmetry Measures for Probability Density Functions

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**Depends** R (>= 3.5.0)

**Imports** stats, sn, skewt, gamlss.dist

## Description

Provides functions and examples for the weak and strong density asymmetry measures in the articles: ``A measure of asymmetry", Patil, Patil and Bagkavos (2012) <[doi:10.1007/s00362-011-0401-6](https://doi.org/10.1007/s00362-011-0401-6)> and ``A measure of asymmetry based on a new necessary and sufficient condition for symmetry", Patil, Bagkavos and Wood (2014) <[doi:10.1007/s13171-013-0034-z](https://doi.org/10.1007/s13171-013-0034-z)>. The measures provided here are useful for quantifying the asymmetry of the shape of a density of a random variable. The package facilitates implementation of the measures which are applicable in a variety of fields including e.g. probability theory, statistics and economics.

**License** GPL (>= 2)

**NeedsCompilation** no

**RoxygenNote** 6.1.0

**LazyData** true

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|          |                                                                 |
|----------|-----------------------------------------------------------------|
| d.sample | <i>Switch between a range of probability density functions.</i> |
|----------|-----------------------------------------------------------------|

---

## Description

Returns the user-specified probability density function out of a range of available options evaluated at selected grid points.

## Usage

```
d.sample(s,dist, p1,p2)
```

## Arguments

|      |                                                                                                      |
|------|------------------------------------------------------------------------------------------------------|
| s    | A scalar or vector: the x-axis grid points where the probability density function will be evaluated. |
| dist | Character string, used as a switch to the user selected distribution function (see details below).   |
| p1   | A scalar. Parameter 1 (vector or object) of the selected density.                                    |
| p2   | A scalar. Parameter 2 (vector or object) of the selected density.                                    |

## Details

Based on user-specified argument `dist`, the function returns the value of the probability density function at `s`.

Supported distributions (along with the corresponding `dist` values) are:

- `weib`: The weibull distribution is implemented as

$$f(s; p_1, p_2) = \frac{p_1}{p_2} \left( \frac{s}{p_2} \right)^{p_1-1} \exp \left\{ - \left( \frac{s}{p_2} \right)^{p_1} \right\}$$

with  $s \geq 0$  where  $p_1$  is the shape parameter and  $p_2$  the scale parameter.

- `lognorm`: The lognormal distribution is implemented as

$$f(s) = \frac{1}{p_2 s \sqrt{2\pi}} e^{-\frac{(\log s - p_1)^2}{2p_2^2}}$$

where  $p_1$  is the mean and  $p_2$  is the standard deviation of the distribution.

- `norm`: The normal distribution is implemented as

$$f(s) = \frac{1}{p_2 \sqrt{2\pi}} e^{-\frac{(s-p_1)^2}{2p_2^2}}$$

where  $p_1$  is the mean and the  $p_2$  is the standard deviation of the distribution.

- `uni`: The uniform distribution is implemented as

$$f(s) = \frac{1}{p_2 - p_1}$$

for  $p_1 \leq s \leq p_2$ .

- `cauchy`: The cauchy distribution is implemented as

$$f(s) = \frac{1}{\pi p_2 \left\{ 1 + \left( \frac{s-p_1}{p_2} \right)^2 \right\}}$$

where  $p_1$  is the location parameter and  $p_2$  the scale parameter.

- `fnorm`: The half normal distribution is implemented as

$$2f(s) - 1$$

where

$$f(s) = \frac{1}{sd\sqrt{2\pi}} e^{-\frac{s^2}{2sd^2}},$$

and  $sd = \sqrt{\pi/2}/p_1$ .

- `normmixt`: The normal mixture distribution is implemented as

$$f(s) = p_1 \frac{1}{p_2[2]\sqrt{2\pi}} e^{-\frac{(s-p_2[1])^2}{2p_2[2]^2}} + (1-p_1) \frac{1}{p_2[4]\sqrt{2\pi}} e^{-\frac{(s-p_2[3])^2}{2p_2[4]^2}}$$

where  $p_1$  is a mixture component (scalar) and  $p_2$  a vector of parameters for the mean and variance of the two mixture components  $p_2 = c(mean1, sd1, mean2, sd2)$ .

- skewnorm: The skew normal distribution with parameter  $p_1$  is implemented as

$$f(s) = 2\phi(s)\Phi(p_1 s)$$

- fas: The Fernandez and Steel distribution is implemented as

$$f(s; p_1, p_2) = \frac{2}{p_1 + \frac{1}{p_1}} \{f_t(s/p_1; p_2)I_{\{s \geq 0\}} + f_t(p_1 s; p_2)I_{\{s < 0\}}\}$$

where  $f_t(x; \nu)$  is the p.d.f. of the  $t$  distribution with  $\nu = 5$  degrees of freedom.  $p_1$  controls the skewness of the distribution with values between  $(0, +\infty)$  and  $p_2$  denotes the degrees of freedom.

- shash: The Sinh-Arcsinh distribution is implemented as

$$f(s; \mu, p_1, p_2, \tau) = \frac{ce^{-r^2/2}}{\sqrt{2\pi}} \frac{1}{p_2} \frac{1}{2} \sqrt{1+z^2}$$

where  $r = \sinh(\sinh(z) - (-p_1))$ ,  $c = \cosh(\sinh(z) - (-p_1))$  and  $z = ((s - \mu)/p_2)$ .  $p_1$  is the vector of skewness,  $p_2$  is the scale parameter,  $\mu = 0$  is the location parameter and  $\tau = 1$  the kurtosis parameter.

## Value

A vector containing the user selected density values at the user specified points  $s$ .

## Author(s)

Dimitrios Bagkavos and Lucia Gamez Gallardo

R implementation and documentation: Dimitrios Bagkavos <dimitrios.bagkavos@gmail.com>, Lucia Gamez Gallardo <gamezgallardolucia@gmail.com>

## References

Bagkavos D., Patil P.N., Wood A.T.A. (2016), A Numerical Study of the Power Function of a New Symmetry Test. In: Cao R., Gonzalez Manteiga W., Romo J. (eds), Nonparametric Statistics. Springer Proceedings in Mathematics and Statistics, vol 175, Springer.

## See Also

[r.sample](#), [q.sample](#), [p.sample](#)

## Examples

```
selected.dens <- "weib" #select Weibull as the density
shape <- 2 # specify shape parameter
scale <- 1 # specify scale parameter
xout <- seq(0.1,5,length=50) #design point where the density is evaluated
d.sample(xout,selected.dens,shape,scale) # calculate density at xout
```

edf

*Empirical cummulative distribution function***Description**

Empirical (nonparametric) cummulative distribution function for given a random sample.

**Usage**

```
edf(xin, xout)
```

**Arguments**

`xin`                    A vector of data points - the available sample.  
`xout`                    A vector of design points where the distribution function will be estimated.

**Details**

The empirical distribution function estimator at  $x$  is defined as the number of observations up to  $x$ , divided by  $n$ , i.e.

$$F_n(x) = \frac{\#\{X_1, \dots, X_n\} \leq x}{n}$$

**Value**

A vector with the estimated distribution function at `xout`.

**Author(s)**

Dimitrios Bagkavos and Lucia Gamez Gallardo

R implementation and documentation:

Dimitrios Bagkavos <dimitrios.bagkavos@gmail.com> , Lucia Gamez Gallardo <gamezgallardolucia@gmail.com>

**References**

Hollander, M. and Wolfe, D.A. (1999), Nonparametric Statistical Methods, 2nd edition, Wiley.

**Examples**

```
x.in <- rexp(200)
x.out <- seq(0.1, 5, length=60)
dist.est <- edf(x.in, x.out)
plot(x.out, dist.est, col="blue", main="Empirical c.d.f.", xlab="x", ylab="probability")
```

---

Epanechnikov

*Epanechnikov kernel*

---

## Description

Implementation of the Epanechnikov kernel.

## Usage

`Epanechnikov(x)`

## Arguments

`x` A vector of data points between  $-\sqrt{5}$  and  $\sqrt{5}$  where the kernel will be evaluated.

## Details

Implements:

$$K(u) = \frac{3}{4\sqrt{5}} \left(1 - \frac{x^2}{5}\right)$$

for  $|x| \leq \sqrt{5}$

## Value

The value of the kernel at  $x$

## Author(s)

Dimitrios Bagkavos and Lucia Gamez Gallardo

R implementation and documentation: Dimitrios Bagkavos <dimitrios.bagkavos@gmail.com> ,  
Lucia Gamez Gallardo <gamezgallardolucia@gmail.com>

## References

[Kernel Statistics](#)

## See Also

[IntEpanechnikov](#)

---

|       |                                                       |
|-------|-------------------------------------------------------|
| eta.s | <i>Strong asymmetry measure <math>\eta(X)</math>.</i> |
|-------|-------------------------------------------------------|

---

## Description

Returns the strong asymmetry measure  $\eta(X)$  of [Patil, Bagkavos and Wood \(2014\)](#).

## Usage

```
eta.s(xin, dist, GridLength, p1, p2)
```

## Arguments

|            |                                                                                                |
|------------|------------------------------------------------------------------------------------------------|
| xin        | A vector of data points - the available sample.                                                |
| dist       | Character string, specifies selected distribution function.                                    |
| GridLength | A non-negative number, which will be rounded up if fractional. Desired length of the sequence. |
| p1         | A scalar. Parameter 1 (vector or object) of the selected distribution.                         |
| p2         | A scalar. Parameter 2 (vector or object) of the selected distribution.                         |

## Details

Implements

$$\eta(X) = -0.5 \operatorname{sign}(\rho_1) \max |\rho_p + \rho_p^*|$$

with  $1/2 \leq p \leq 1$ .

Uses maximum likelihood estimates for the unknown functionals in the definition of the measure.

## Value

Returns a scalar, the value of the strong asymmetry measure  $\eta(X)$ .

## Author(s)

Dimitrios Bagkavos and Lucia Gamez Gallardo

R implementation and documentation: Dimitrios Bagkavos <dimitrios.bagkavos@gmail.com> ,  
Lucia Gamez Gallardo <gamezgallardolucia@gmail.com>

## References

- [Patil P.N., Bagkavos D. and Wood A.T.A., \(2014\). A measure of asymmetry based on a new necessary and sufficient condition for symmetry, Sankhya A, 76, 123–145.](#)
- [Bagkavos D., Patil P.N., Wood A.T.A. \(2016\), A Numerical Study of the Power Function of a New Symmetry Test. In: Cao R., Gonzalez Manteiga W., Romo J. \(eds\), Nonparametric Statistics. Springer Proceedings in Mathematics and Statistics, vol 175, Springer.](#)

**See Also**

[eta.w.hat.bc](#), [eta.w.hat](#), [eta.w.breve](#), [eta.w.breve.bc](#), [eta.w.tilde](#), [eta.w.tilde.bc](#)

**Examples**

```
selected.dist <- "norm" #select norm as the distribution
m.use <- mean(GDP.Per.head.dist.2005)
sd.use<- sd(GDP.Per.head.dist.2005)
grid <- 50

s.use<- GDP.Per.head.dist.1995
eta.s(GDP.Per.head.dist.2005,selected.dist,grid,m.use,sd.use)
```

---

eta.s.exact

---

Strong asymmetry measure  $\eta(X)$ .

---

**Description**

Returns the strong asymmetry measure  $\eta(X)$  of [Patil, Bagkavos and Wood \(2014\)](#).

**Usage**

```
eta.s.exact(xin, dist, GridLength, p1, p2)
```

**Arguments**

|            |                                                                                               |
|------------|-----------------------------------------------------------------------------------------------|
| xin        | A vector of data points - the available sample.                                               |
| dist       | Character string, specifies selected distribution function.                                   |
| GridLength | A non-negative number, which will be rounded up if fractional.Desired length of the sequence. |
| p1         | A scalar. Parameter 1 (vector or object) of the selected distribution.                        |
| p2         | A scalar. Parameter 2 (vector or object) of the selected distribution.                        |

**Details**

Implements

$$\eta(X) = -0.5 \text{sign}(\rho_1) \max |\rho_p + \rho_p^*|$$

with  $1/2 \leq p \leq 1$  This version uses exact p.d.f. and c.d.f. evaluation and not estimates of the unknown functionals.

**Value**

Returns a scalar, the value of the strong asymmetry measure  $\eta(X)$ .

**Author(s)**

Dimitrios Bagkavos and Lucia Gamez Gallardo

R implementation and documentation: Dimitrios Bagkavos <dimitrios.bagkavos@gmail.com> ,  
Lucia Gamez Gallardo <gamezgallardolucia@gmail.com>

**References**

- Patil P.N., Bagkavos D. and Wood A.T.A., (2014). A measure of asymmetry based on a new necessary and sufficient condition for symmetry, *Sankhya A*, 76, 123–145.
- Bagkavos D., Patil P.N., Wood A.T.A. (2016), A Numerical Study of the Power Function of a New Symmetry Test. In: Cao R., González Manteiga W., Romo J. (eds) *Nonparametric Statistics. Springer Proceedings in Mathematics and Statistics*, vol 175, Springer.

**See Also**

[eta.w.hat.bc](#), [eta.w.hat](#), [eta.w.breve](#), [eta.w.breve.bc](#), [eta.w.tilde](#), [eta.w.tilde.bc](#), [eta.s](#)

**Examples**

```
selected.dist <- "norm" #select norm as the distribution
m.use <- 2
sd.use<- 2
grid <- 50
s.use<- rnorm(100)
eta.s.exact(s.use,selected.dist,grid,m.use,sd.use) # calculate eta.s at xout
```

---

eta.w.breve

---

*Asymmetry coefficient  $\check{\eta}$* 


---

**Description**

Implements the asymmetry coefficient  $\check{\eta}$  of [Patil, Patil and Bagkavos \(2012\)](#).

**Usage**

```
eta.w.breve(xin, kfun)
```

**Arguments**

|      |                                                 |
|------|-------------------------------------------------|
| xin  | A vector of data points - the available sample. |
| kfun | The kernel to use in the density estimate.      |

### Details

Given a sample  $X_1, X_2, \dots, X_n$  from a continuous density function  $f(x)$  and distribution function  $F(x)$ ,  $\check{\eta}$  is defined by

$$\check{\eta} = - \frac{\sum_{i=1}^n U_i W_i - n \bar{U} \bar{W}}{\sqrt{(\sum_{i=1}^n U_i^2 - n \bar{U}^2)(\sum_{i=1}^n W_i^2 - n \bar{W}^2)}}$$

where

$$U_i = \hat{f}(X_i), \quad W_i = F_n(X_i), \quad \bar{U} = n^{-1} \sum_{i=1}^n U_i, \quad \bar{W} = n^{-1} \sum_{i=1}^n W_i.$$

### Value

Returns a scalar, the estimate of  $\check{\eta}$ .

### Author(s)

Dimitrios Bagkavos and Lucia Gamez Gallardo

R implementation and documentation: Dimitrios Bagkavos <dimitrios.bagkavos@gmail.com>, Lucia Gamez Gallardo <gamezgallardolucia@gmail.com>

### References

Patil, P.N., Patil, P.P. and Bagkavos, D., (2012), A measure of asymmetry. Stat. Papers, 53, 971–985.

### See Also

[eta.w.hat.bc](#), [eta.w.hat](#), [eta.w.breve.bc](#), [eta.w.tilde](#), [eta.w.tilde.bc](#)

### Examples

```
eta.w.breve(GDP.Per.head.dist.1995,Epanechnikov)
0.329707 #estimate of etabreve
```

eta.w.breve.bc

*Asymmetry coefficient  $\check{\eta}$  using boundary correction***Description**

Implements the asymmetry coefficient  $\check{\eta}$  of [Patil, Patil and Bagkavos \(2012\)](#).

**Usage**

```
eta.w.breve.bc(xin, kfun)
```

**Arguments**

`xin` A vector of data points - the available sample.  
`kfun` The kernel to use in the density estimate.

**Details**

Given a sample  $X_1, X_2, \dots, X_n$  from a continuous density function  $f(x)$  and distribution function  $F(x)$ .  $\check{\eta}$  is defined by

$$\check{\eta} = - \frac{\sum_{i=1}^n U_i W_i - n \bar{U} \bar{W}}{\sqrt{(\sum_{i=1}^n U_i^2 - n \bar{U}^2)(\sum_{i=1}^n W_i^2 - n \bar{W}^2)}}$$

where

$$U_i = \hat{f}(X_i), W_i = F_n(X_i), \bar{U} = n^{-1} \sum_{i=1}^n U_i, \bar{W} = n^{-1} \sum_{i=1}^n W_i.$$

eta.w.breve.bc uses reflection to correct the boundary bias of the kernel density estimate kde

**Value**

Returns a scalar, the estimate of  $\check{\eta}$ .

**Author(s)**

Dimitrios Bagkavos and Lucia Gamez Gallardo

R implementation and documentation: Dimitrios Bagkavos <dimitrios.bagkavos@gmail.com> ,  
 Lucia Gamez Gallardo <gamezgallardolucia@gmail.com>

**References**

[Patil, P.N., Patil, P.P. and Bagkavos, D., \(2012\), A measure of asymmetry. Stat. Papers, 53, 971-985.](#)

**See Also**

[eta.w.hat.bc](#), [eta.w.hat](#), [eta.w.breve](#), [eta.w.tilde](#), [eta.w.tilde.bc](#)

**Examples**

```
eta.w.breve.bc(GDP.Per.head.dist.1995,Epanechnikov)
0.329707 #estimate of etabreve
```

---

|           |                                                      |
|-----------|------------------------------------------------------|
| eta.w.hat | <i>Asymmetry coefficient <math>\hat{\eta}</math></i> |
|-----------|------------------------------------------------------|

---

**Description**

Implements the asymmetry coefficient  $\hat{\eta}$  of [Patil, Patil and Bagkavos \(2012\)](#).

**Usage**

```
eta.w.hat(xin, kfun)
```

**Arguments**

`xin`                      A vector of data points - the available sample.  
`kfun`                     The kernel to use in the density estimate.

**Details**

Given a sample  $X_1, X_2, \dots, X_n$  from a continuous density function  $f(x)$  and distribution function  $F(x)$ ,  $\hat{\eta}$  is defined by

$$\hat{\eta} = - \frac{\sum_{i=1}^n U_i V_i - n \bar{U} \bar{V}}{\sqrt{(\sum_{i=1}^n U_i^2 - n \bar{U}^2)(\sum_{i=1}^n V_i^2 - n \bar{V}^2)}}$$

where

$$U_i = \hat{f}(X_i), \quad V_i = \hat{F}(X_i), \quad \bar{U} = n^{-1} \sum_{i=1}^n U_i, \quad \bar{V} = n^{-1} \sum_{i=1}^n V_i.$$

**Value**

Returns a scalar, the estimate of  $\hat{\eta}$ .

**Author(s)**

Dimitrios Bagkavos and Lucia Gamez Gallardo

R implementation and documentation: Dimitrios Bagkavos <dimitrios.bagkavos@gmail.com>, Lucia Gamez Gallardo <gamezgallardolucia@gmail.com>

## References

Patil, P.N., Patil, P.P. and Bagkavos, D., (2012), A measure of asymmetry. Stat. Papers, 53, 971-985.

## See Also

[eta.w.hat.bc](#), [eta.w.breve](#), [eta.w.breve.bc](#), [eta.w.tilde](#), [eta.w.tilde.bc](#)

## Examples

```
eta.w.hat(GDP.Per.head.dist.1995,Epanechnikov)
0.3463025 #estimate of etahat
```

---

|              |                                                              |
|--------------|--------------------------------------------------------------|
| eta.w.hat.bc | Asymmetry coefficient $\hat{\eta}$ using boundary correction |
|--------------|--------------------------------------------------------------|

---

## Description

Implements the asymmetry coefficient  $\hat{\eta}$  of [Patil, Patil and Bagkavos \(2012\)](#)

## Usage

```
eta.w.hat.bc(xin, kfun)
```

## Arguments

`xin`                      A vector of data points - the available sample.  
`kfun`                     The kernel to use in the density estimate.

## Details

Given a sample  $X_1, X_2, \dots, X_n$  from a continuous density function  $f(x)$  and distribution function  $F(x)$ ,  $\hat{\eta}$  is defined by

$$\hat{\eta} = - \frac{\sum_{i=1}^n U_i V_i - n \bar{U} \bar{V}}{\sqrt{(\sum_{i=1}^n U_i^2 - n \bar{U}^2)(\sum_{i=1}^n V_i^2 - n \bar{V}^2)}}$$

where

$$U_i = \hat{f}(X_i), \quad V_i = \hat{F}(X_i), \quad \bar{U} = n^{-1} \sum_{i=1}^n U_i, \quad \bar{V} = n^{-1} \sum_{i=1}^n V_i.$$

`eta.w.hat.bc` uses reflection to correct the boundary bias issue of the kernel estimate `kde`.

**Value**

Returns a scalar, the estimate of  $\hat{\eta}$ .

**Author(s)**

Dimitrios Bagkavos and Lucia Gamez Gallardo

R implementation and documentation: Dimitrios Bagkavos <dimitrios.bagkavos@gmail.com>, Lucia Gamez Gallardo <gamezgallardolucia@gmail.com>

**References**

Patil, P.N., Patil, P.P. and Bagkavos, D., (2012), A measure of asymmetry. Stat. Papers, 53, 971-985.

**See Also**

[eta.w.hat](#), [eta.w.breve](#), [eta.w.breve.bc](#), [eta.w.tilde](#), [eta.w.tilde.bc](#)

**Examples**

```
eta.w.hat.bc(GDP.Per.head.dist.1995,Epanechnikov)
0.3463025 #estimate of etahat.bc
```

---

eta.w.tilde

*Asymmetry coefficient  $\tilde{\eta}$*

---

**Description**

Implements the asymmetry coefficient  $\tilde{\eta}$  of [Patil, Patil and Bagkavos \(2012\)](#).

**Usage**

```
eta.w.tilde(xin, kfun)
```

**Arguments**

**xin**                      A vector of data points - the available sample.  
**kfun**                    The kernel to use in the density estimate.

**Details**

Given a sample  $X_1, X_2, \dots, X_n$  from a continuous density function  $f(x)$  and distribution function  $F(x)$ .  $\tilde{\eta}$  is defined by

$$\tilde{\eta} = -\frac{\sum_{i=1}^n U_i V_i - (n/2)\bar{U}}{\sqrt{(n/12)(\sum_{i=1}^n U_i^2 - n\bar{U}^2)}}$$

where

$$U_i = \hat{f}(X_i), \quad V_i = F(X_i), \quad \bar{U} = n^{-1} \sum_{i=1}^n U_i, \quad \bar{V} = n^{-1} \sum_{i=1}^n V_i.$$

**Value**

Returns a scalar, the estimate of  $\tilde{\eta}$ .

**Author(s)**

Dimitrios Bagkavos and Lucia Gamez Gallardo

R implementation and documentation: Dimitrios Bagkavos <dimitrios.bagkavos@gmail.com> ,  
Lucia Gamez Gallardo <gamezgallardolucia@gmail.com>

**References**

Patil, P.N., Patil, P.P. and Bagkavos, D., (2012), A measure of asymmetry. Stat. Papers, 53, 971-985.

**See Also**

[eta.w.hat.bc](#), [eta.w.hat](#), [eta.w.breve.bc](#), [eta.w.breve](#), [eta.w.tilde.bc](#)

**Examples**

```
eta.w.tilde(GDP.Per.head.dist.1995,Epanechnikov)
0.3333485 #estimate of etatile
```

---

|                |                                                                                  |
|----------------|----------------------------------------------------------------------------------|
| eta.w.tilde.bc | <i>Asymmetry coefficient <math>\tilde{\eta}</math> using boundary correction</i> |
|----------------|----------------------------------------------------------------------------------|

---

**Description**

Implements the asymmetry coefficient  $\tilde{\eta}$  of Patil, Patil and Bagkavos (2012).

**Usage**

```
eta.w.tilde.bc(xin, kfun)
```

**Arguments**

|      |                                                 |
|------|-------------------------------------------------|
| xin  | A vector of data points - the available sample. |
| kfun | The kernel to use in the density estimate.      |

**Details**

Given a sample  $X_1, X_2, \dots, X_n$  from a continuous density function  $f(x)$  and distribution function  $F(x)$ ,  $\tilde{\eta}$  is defined by

$$\tilde{\eta} = -\frac{\sum_{i=1}^n U_i V_i - (n/2)\bar{U}}{\sqrt{(n/12)(\sum_{i=1}^n U_i^2 - n\bar{U}^2)}}$$

where

$$U_i = \hat{f}(X_i), \quad V_i = F(X_i), \quad \bar{U} = n^{-1} \sum_{i=1}^n U_i, \quad \bar{V} = n^{-1} \sum_{i=1}^n V_i.$$

`eta.w.tilde.bc` uses reflection to correct the boundary bias of `kde`.

**Value**

Returns a scalar, the estimate of  $\tilde{\eta}$ .

**Author(s)**

Dimitrios Bagkavos and Lucia Gamez Gallardo

R implementation and documentation: Dimitrios Bagkavos <dimitrios.bagkavos@gmail.com> ,  
Lucia Gamez Gallardo <gamezgallardolucia@gmail.com>

**References**

Patil, P.N., Patil, P.P. and Bagkavos, D., (2012), A measure of asymmetry. *Stat. Papers*, 53, 971-985.

**See Also**

[eta.w.hat.bc](#), [eta.w.hat](#), [eta.w.breve.bc](#), [eta.w.breve](#), [eta.w.tilde](#)

**Examples**

```
eta.w.tilde.bc(GDP.Per.head.dist.1995,Epanechnikov)
0.3333485 #estimate of etatile.bc
```

---

GDP.Per.head.dist.1995

*annual Gross Domestic Product (GDP) per head across 15 European Union (EU) countries*

---

**Description**

Contains values of the GDP/head distribution of 216 EU regions (the so called NUTS-2 level of the Eurostat categorization of territories within the EU for the year 1995).

**Usage**

GDP.Per.head.dist.1995

**Format**

A vector with 184 values of the GDP/head distribution for 1995.

**Source**

Monfort, P. (2008). Convergence of EU regions measures and evolution. EU short papers on regional research and indicators, Directorate-General for Regional Policy 1/2008.

**References**

Monfort, P. (2008). Convergence of EU regions measures and evolution. EU short papers on regional research and indicators

**See Also**

[GDP.Per.head.dist.2005](#)

---

GDP.Per.head.dist.2005

*annual Gross Domestic Product (GDP) per head across 15 European Union (EU) countries*

---

**Description**

Contains values of the GDP/head distribution of 216 EU regions (the so called NUTS-2 level of the Eurostat categorization of territories within the EU for the year 2005).

**Usage**

GDP.Per.head.dist.1995

**Format**

A vector with 184 values of the GDP/head distribution for 2005.

**Source**

Monfort, P. (2008). Convergence of EU regions measures and evolution. EU short papers on regional research and indicators, Directorate-General for Regional Policy 1/2008.

**References**

Monfort, P. (2008). Convergence of EU regions measures and evolution. EU short papers on regional research and indicators

**See Also**

[GDP.Per.head.dist.1995](#)

---

|                 |                                         |
|-----------------|-----------------------------------------|
| IntEpanechnikov | <i>Integrated Epanechnikov function</i> |
|-----------------|-----------------------------------------|

---

**Description**

Implements the Integrated Epanechnikov kernel.

**Usage**

```
IntEpanechnikov(x)
```

**Arguments**

`x` A vector of design points with values from  $-\sqrt{5}$  to  $\sqrt{5}$ .

**Details**

Implements:

$$K(u) = \int_{-\infty}^u \frac{3}{4\sqrt{5}} \left(1 - \frac{x^2}{5}\right) dx$$

for  $|x| \leq \sqrt{5}$

**Value**

The value of the integrated kernel function at the user designated points.

**Author(s)**

Dimitrios Bagkavos and Lucia Gamez Gallardo

R implementation and documentation: Dimitrios Bagkavos <dimitrios.bagkavos@gmail.com> ,  
 Lucia Gamez Gallardo <gamezgallardolucia@gmail.com>

**References**

[Kernel Statistics](#)

**See Also**

[Epanechnikov](#)

---

|        |                                            |
|--------|--------------------------------------------|
| IntKde | <i>Integrated Kernel density estimator</i> |
|--------|--------------------------------------------|

---

**Description**

Classical univariate integrated kernel density estimator

**Usage**

```
IntKde(xin, xout, h, kfun)
```

**Arguments**

|      |                                                                                                    |
|------|----------------------------------------------------------------------------------------------------|
| xin  | A vector of data points - the available sample size.                                               |
| xout | grid points where the distribution function will be estimated.                                     |
| h    | The bandwidth parameter. Defaults to $3.572 * \sigma * n^{-1/3}$ according to Bowman et al.(1998). |
| kfun | The kernel to use in the distribution function estimate.                                           |

**Details**

It implements the classical density integrated kernel estimator.

Let  $X_1, X_2, \dots, X_n$  be a univariate independent and identically distributed sample drawn from some unknown distribution function  $F$ . Its kernel density estimator is

$$\hat{F}(x) = n^{-1} \sum_{i=1}^n K \{ (x - X_i) h^{-1} \}$$

where  $K$  is an integrated kernel, and  $h > 0$  is a smoothing parameter called the bandwidth.

**Value**

Returns a vector with the estimate of the distribution function at the user specified grid points.

**Author(s)**

Dimitrios Bagkavos and Lucia Gamez Gallardo

R implementation and documentation: Dimitrios Bagkavos <dimitrios.bagkavos@gmail.com>, Lucia Gamez Gallardo <gamezgallardolucia@gmail.com>

**References**

Bowman, A., Hall, P., and Prvan, T., (1998), Bandwidth Selection for the Smoothing of Distribution Functions, *Biometrika*, 799-808.

**See Also**

[bw.nrd](#), [bw.nrd0](#), [bw.ucv](#), [bw.bcv](#)

**Examples**

```
x.in <- rnorm(100)
x.out <- seq(-3.4, 3.4, length=60)
kernel <- IntEpanechnikov
dist.est <- IntKde(xin=x.in, xout=x.out, kfun=kernel)
plot(x.out, dist.est, type="l", col="red", main="Kernel c.d.f. estimator")
```

---

|     |                                  |
|-----|----------------------------------|
| kde | <i>Kernel density estimator.</i> |
|-----|----------------------------------|

---

**Description**

Classical univariate kernel density estimator.

**Usage**

```
kde(xin, xout, h, kfun)
```

**Arguments**

|      |                                                                                 |
|------|---------------------------------------------------------------------------------|
| xin  | A vector of data points. Missing values not allowed.                            |
| xout | A vector of grid points at which the estimate will be calculated.               |
| h    | A scalar, the bandwidth to use in the estimate, e.g. <code>bw.nrd(xin)</code> . |
| kfun | Kernel function to use.                                                         |

**Details**

Implements the classical density kernel estimator based on a sample  $X_1, X_2, \dots, X_n$  of i.i.d observations from a distribution  $F$  with density  $h$ . The estimator is defined by

$$\hat{f}(x) = n^{-1} \sum_{i=1}^n K_h(x - X_i)$$

where  $h$  is determined by a bandwidth selector such as Silverman's default plug-in rule and  $K$ , the kernel, is a non-negative probability density function.

**Value**

A vector with the density estimates at the designated points `xout`.

**Author(s)**

Dimitrios Bagkavos and Lucia Gamez Gallardo

R implementation and documentation: Dimitrios Bagkavos <dimitrios.bagkavos@gmail.com> ,  
Lucia Gamez Gallardo <gamezgallardolucia@gmail.com>

**References**

Silverman, B.W. (1986), *Density Estimation for Statistics and Data Analysis*, Chapman and Hall, London.

**See Also**

[bw.nrd](#), [bw.nrd0](#), [bw.ucv](#), [bw.bcv](#)

**Examples**

```
x.in <- rnorm(100)
x.out <- seq(-3.4,3.4,length=60)
bandwidth <- bw.nrd(x.in)
kernel <- Epanechnikov
dens.est <- kde(x.in,x.out,bandwidth,kernel)
plot(x.out,dens.est,col="red",main="Kernel density estimator")
```

---

p.sample

*Switch between a range of available cumulative distribution functions.*

---

**Description**

Returns the value of the selected cumulative distribution function at user supplied grid points.

**Usage**

```
p.sample(s,dist, p1,p2)
```

**Arguments**

|      |                                                                                                           |
|------|-----------------------------------------------------------------------------------------------------------|
| s    | A scalar or vector: the x-axis grid points where the cumulative distribution function is to be evaluated. |
| dist | Character string, used as a switch to the user selected distribution function (see details below).        |
| p1   | A scalar. Parameter 1 (vector or object) of the selected distribution.                                    |
| p2   | A scalar. Parameter 2 (vector or object) of the selected distribution.                                    |

### Details

Based on the user-specified argument `dist`, the function returns the value of the cumulative distribution function at `s`.

Supported distributions (along with the corresponding `dist` values) are:

- `weib`: The Weibull distribution is implemented as

$$F(s) = 1 - \exp \left\{ - \left( \frac{s}{p_2} \right)^{p_1} \right\}$$

with  $s > 0$  where  $p_1$  is the shape parameter and  $p_2$  the scale parameter.

- `lognorm`: The lognormal distribution is implemented as

$$F(s) = \Phi \left( \frac{\ln s - p_1}{p_2} \right)$$

where  $p_1$  is the mean,  $p_2$  is the standard deviation and  $\Phi$  is the cumulative distribution function of the standard normal distribution.

- `norm`: The normal distribution is implemented as

$$\Phi(s) = \frac{1}{\sqrt{2\pi}p_2} \int_{-\infty}^s e^{-\frac{(t-p_1)^2}{2p_2^2}} dt$$

where  $p_1$  is the mean and the  $p_2$  is the standard deviation.

- `uni`: The uniform distribution is implemented as

$$F(s) = \frac{s - p_1}{p_2 - p_1}$$

for  $p_1 \leq s \leq p_2$ .

- `cauchy`: The cauchy distribution is implemented as

$$F(s; p_1, p_2) = \frac{1}{\pi} \arctan \left( \frac{s - p_1}{p_2} \right) + \frac{1}{2}$$

where  $p_1$  is the location parameter and  $p_2$  the scale parameter.

- `fnorm`: The half normal distribution is implemented as

$$F_S(s; \sigma) = \int_0^s \frac{\sqrt{2/\pi}}{\sigma} \exp \left\{ -\frac{x^2}{2\sigma^2} \right\} dx$$

where  $mean = 0$  and  $sd = \sqrt{\pi/2}/p_1$ .

- `normmixt`: The normal mixture distribution is implemented as

$$F(s) = p_1 \frac{1}{p_2[2]\sqrt{2\pi}} \int_{-\infty}^s e^{-\frac{(t-p_2[1])^2}{2p_2[2]^2}} dt + (1 - p_1) \frac{1}{p_2[4]\sqrt{2\pi}} \int_{-\infty}^s e^{-\frac{(t-p_2[3])^2}{2p_2[4]^2}} dt$$

where  $p_1$  is a mixture component (scalar) and  $p_2$  a vector of parameters for the mean and variance of the two mixture components  $p_2 = c(mean1, sd1, mean2, sd2)$ .

- skewnorm: The skew normal distribution is implemented as

$$F(y; p_1) = \Phi\left(\frac{y - \xi}{\omega}\right) - 2T\left(\frac{y - \xi}{\omega}, p_1\right)$$

where  $location = \xi = 0$ ,  $scale = \omega = 1$ ,  $parameter = p_1$  and  $T(h, a)$  is the Owens T function, defined by

$$T(h, a) = \frac{1}{2\pi} \int_0^a \exp\left\{\frac{-0.5h^2(1+x^2)}{1+x^2}\right\} dx, -\infty \leq h, a \leq \infty$$

- fas: The Fernandez and Steel distribution is implemented as

$$F(s; p_1, p_2) = \frac{2}{p_1 + \frac{1}{p_1}} \left\{ \int_{-\infty}^s f_t(x/p_1; p_2) I_{\{x \geq 0\}} dx + \int_{-\infty}^s f_t(p_1 x; p_2) I_{\{x < 0\}} dx \right\}$$

where  $f_t(x; \nu)$  is the p.d.f. of the t distribution with  $\nu = 5$  degrees of freedom.  $p_1$  controls the skewness of the distribution with values between  $(0, +\infty)$  and  $p_2$  is the degrees of freedom.

- shash: The Sinh-Arcsinh distribution is implemented as

$$F(s; \mu, p_2, p_1, \tau) = \int_{-\infty}^s \frac{ce^{-r^2/2}}{\sqrt{2\pi}} \frac{1}{p_2} \frac{1}{2} \sqrt{1+z^2} dz$$

where  $r = \sinh(\sinh(z) - p_1)$ ,  $c = \cosh(\sinh(z) - p_1)$  and  $z = (s - \mu)/p_2$ .  $p_1$  is the vector of skewness,  $p_2$  is the scale parameter,  $\mu = 0$  is the location parameter and  $\tau = 1$  the kurtosis parameter.

## Value

A vector containing the cumulative distribution function values at the user specified points  $s$ .

## Author(s)

Dimitrios Bagkavos and Lucia Gamez Gallardo

R implementation and documentation: Dimitrios Bagkavos <dimitrios.bagkavos@gmail.com> ,  
Lucia Gamez Gallardo <gamezgallardolucia@gmail.com>

## References

Bagkavos D., Patil P.N., Wood A.T.A. (2016), A Numerical Study of the Power Function of a New Symmetry Test. In: Cao R., Gonzalez Manteiga W., Romo J. (eds) Nonparametric Statistics. Springer Proceedings in Mathematics and Statistics, vol 175, Springer.

## See Also

[r.sample](#), [q.sample](#), [d.sample](#)

### Examples

```
selected.d <- "weib" #select Weibull as the CDF
shape <- 2 # specify shape parameter
scale <- 1 # specify scale parameter
xout <- seq(0.1,5,length=50) #design point where the CDF is evaluated
p.sample(xout,selected.d,shape,scale) # calculate CDF at xout
```

pdfsq

*Calculate  $f^2(x)$* 

### Description

Calculates the square of a density.

### Usage

```
pdfsq(s,dist, p1,p2)
```

### Arguments

|      |                                                                                                      |
|------|------------------------------------------------------------------------------------------------------|
| s    | A scalar or vector: the x-axis grid points where the probability density function will be evaluated. |
| dist | Character string, used as a switch to the user selected distribution function (see details below).   |
| p1   | A scalar. Parameter 1 (vector or object) of the selected density.                                    |
| p2   | A scalar. Parameter 2 (vector or object) of the selected density.                                    |

### Details

Based on user-specified argument dist, the function returns the value of  $f^2(x)dx$ , used in the definitions of  $\rho_p^*$ ,  $\rho_p$  and their exact versions.

Supported distributions (along with the corresponding dist values) are:

- weib: The weibull distribution is implemented as

$$f(s; p_1, p_2) = \frac{p_1}{p_2} \left( \frac{s}{p_2} \right)^{p_1-1} \exp \left\{ - \left( \frac{s}{p_2} \right)^{p_1} \right\}$$

with  $s \geq 0$  where  $p_1$  is the shape parameter and  $p_2$  the scale parameter.

- lognorm: The lognormal distribution is implemented as

$$f(s) = \frac{1}{p_2 s \sqrt{2\pi}} e^{-\frac{(\log s - p_1)^2}{2p_2^2}}$$

where  $p_1$  is the mean and  $p_2$  is the standard deviation of the distribution.

- norm: The normal distribution is implemented as

$$f(s) = \frac{1}{p_2\sqrt{2\pi}} e^{-\frac{(s-p_1)^2}{2p_2^2}}$$

where  $p_1$  is the mean and the  $p_2$  is the standard deviation of the distribution.

- uni: The uniform distribution is implemented as

$$f(s) = \frac{1}{p_2 - p_1}$$

for  $p_1 \leq s \leq p_2$ .

- cauchy: The cauchy distribution is implemented as

$$f(s) = \frac{1}{\pi p_2 \left\{ 1 + \left( \frac{s-p_1}{p_2} \right)^2 \right\}}$$

where  $p_1$  is the location parameter and  $p_2$  the scale parameter.

- fnorm: The half normal distribution is implemented as

$$2f(s) - 1$$

where

$$f(s) = \frac{1}{sd\sqrt{2\pi}} e^{-\frac{s^2}{2sd^2}},$$

and  $sd = \sqrt{\pi/2}/p_1$ .

- normmixt: The normal mixture distribution is implemented as

$$f(s) = p_1 \frac{1}{p_2[2]\sqrt{2\pi}} e^{-\frac{(s-p_2[1])^2}{2p_2[2]^2}} + (1-p_1) \frac{1}{p_2[4]\sqrt{2\pi}} e^{-\frac{(s-p_2[3])^2}{2p_2[4]^2}}$$

where  $p_1$  is a mixture component (scalar) and  $p_2$  a vector of parameters for the mean and variance of the two mixture components  $p_2 = c(mean1, sd1, mean2, sd2)$ .

- skewnorm: The skew normal distribution with parameter  $p_1$  is implemented as

$$f(s) = 2\phi(s)\Phi(p_1 s)$$

.

- fas: The Fernandez and Steel distribution is implemented as

$$f(s; p_1, p_2) = \frac{2}{p_1 + \frac{1}{p_1}} \left\{ f_t(s/p_1; p_2) I_{\{s \geq 0\}} + f_t(p_1 s; p_2) I_{\{s < 0\}} \right\}$$

where  $f_t(x; \nu)$  is the p.d.f. of the  $t$  distribution with  $\nu = 5$  degrees of freedom.  $p_1$  controls the skewness of the distribution with values between  $(0, +\infty)$  and  $p_2$  denotes the degrees of freedom.

- shash: The Sinh-Arcsinh distribution is implemented as

$$f(s; \mu, p_1, p_2, \tau) = \frac{ce^{-r^2/2}}{\sqrt{2\pi}} \frac{1}{p_2} \frac{1}{2} \sqrt{1+z^2}$$

where  $r = \sinh(\sinh(z) - (-p_1))$ ,  $c = \cosh(\sinh(z) - (-p_1))$  and  $z = ((s - \mu)/p_2)$ .  $p_1$  is the vector of skewness,  $p_2$  is the scale parameter,  $\mu = 0$  is the location parameter and  $\tau = 1$  the kurtosis parameter.

**Value**

A vector containing the user selected density values at the user specified points  $s$ .

**Author(s)**

Dimitrios Bagkavos and Lucia Gamez Gallardo

R implementation and documentation: Dimitrios Bagkavos <dimitrios.bagkavos@gmail.com>, Lucia Gamez Gallardo <gamezgallardolucia@gmail.com>

**References**

Bagkavos D., Patil P.N., Wood A.T.A. (2016), A Numerical Study of the Power Function of a New Symmetry Test. In: Cao R., Gonzalez Manteiga W., Romo J. (eds) Nonparametric Statistics. Springer Proceedings in Mathematics and Statistics, vol 175, Springer.

**See Also**

[r.sample](#), [q.sample](#), [p.sample](#)

**Examples**

```
selected.dens <- "weib" #select Weibull
shape <- 2 # specify shape parameter
scale <- 1 # specify scale parameter
xout <- seq(0.1,5,length=50) #design point
pdfsq(xout,selected.dens,shape,scale) # calculate the square density at xout
```

---

pdfsqcdf

*Calculate  $f^2(x)F(x)$*

---

**Description**

Return the product  $f^2(x)F(x)$

**Usage**

```
pdfsqcdf(s,dist, p1,p2)
```

**Arguments**

|        |                                                                                                      |
|--------|------------------------------------------------------------------------------------------------------|
| $s$    | A scalar or vector: the x-axis grid points where the probability density function will be evaluated. |
| $dist$ | Character string, used as a switch to the user selected distribution function (see details below).   |
| $p1$   | A scalar. Parameter 1 (vector or object) of the selected density.                                    |
| $p2$   | A scalar. Parameter 2 (vector or object) of the selected density.                                    |

## Details

Based on user-specified argument `dist`, the function returns the value of  $f^2(x)F(x)dx$ , used in the definitions of  $\rho_p^*$ ,  $\rho_p$  and their exact versions.

Supported distributions (along with the corresponding `dist` values) are:

- `weib`: The weibull distribution is implemented as

$$f(s; p_1, p_2) = \frac{p_1}{p_2} \left( \frac{s}{p_2} \right)^{p_1-1} \exp \left\{ - \left( \frac{s}{p_2} \right)^{p_1} \right\}$$

with  $s \geq 0$  where  $p_1$  is the shape parameter and  $p_2$  the scale parameter.

- `lognorm`: The lognormal distribution is implemented as

$$f(s) = \frac{1}{p_2 s \sqrt{2\pi}} e^{-\frac{(\log s - p_1)^2}{2p_2^2}}$$

where  $p_1$  is the mean and  $p_2$  is the standard deviation of the distribution.

- `norm`: The normal distribution is implemented as

$$f(s) = \frac{1}{p_2 \sqrt{2\pi}} e^{-\frac{(s-p_1)^2}{2p_2^2}}$$

where  $p_1$  is the mean and the  $p_2$  is the standard deviation of the distribution.

- `uni`: The uniform distribution is implemented as

$$f(s) = \frac{1}{p_2 - p_1}$$

for  $p_1 \leq s \leq p_2$ .

- `cauchy`: The cauchy distribution is implemented as

$$f(s) = \frac{1}{\pi p_2 \left\{ 1 + \left( \frac{s-p_1}{p_2} \right)^2 \right\}}$$

where  $p_1$  is the location parameter and  $p_2$  the scale parameter.

- `fnorm`: The half normal distribution is implemented as

$$2f(s) - 1$$

where

$$f(s) = \frac{1}{sd\sqrt{2\pi}} e^{-\frac{s^2}{2sd^2}},$$

and  $sd = \sqrt{\pi/2}/p_1$ .

- `normmixt`: The normal mixture distribution is implemented as

$$f(s) = p_1 \frac{1}{p_2[2]\sqrt{2\pi}} e^{-\frac{(s-p_2[1])^2}{2p_2[2]^2}} + (1-p_1) \frac{1}{p_2[4]\sqrt{2\pi}} e^{-\frac{(s-p_2[3])^2}{2p_2[4]^2}}$$

where  $p_1$  is a mixture component (scalar) and  $p_2$  a vector of parameters for the mean and variance of the two mixture components  $p_2 = c(mean1, sd1, mean2, sd2)$ .

- skewnorm: The skew normal distribution with parameter  $p_1$  is implemented as

$$f(s) = 2\phi(s)\Phi(p_1 s)$$

.

- fas: The Fernandez and Steel distribution is implemented as

$$f(s; p_1, p_2) = \frac{2}{p_1 + \frac{1}{p_1}} \{f_t(s/p_1; p_2)I_{\{s \geq 0\}} + f_t(p_1 s; p_2)I_{\{s < 0\}}\}$$

where  $f_t(x; \nu)$  is the p.d.f. of the  $t$  distribution with  $\nu = 5$  degrees of freedom.  $p_1$  controls the skewness of the distribution with values between  $(0, +\infty)$  and  $p_2$  denotes the degrees of freedom.

- shash: The Sinh-Arcsinh distribution is implemented as

$$f(s; \mu, p_1, p_2, \tau) = \frac{ce^{-r^2/2}}{\sqrt{2\pi}} \frac{1}{p_2} \frac{1}{2} \sqrt{1+z^2}$$

where  $r = \sinh(\sinh(z) - (-p_1))$ ,  $c = \cosh(\sinh(z) - (-p_1))$  and  $z = ((s - \mu)/p_2)$ .  $p_1$  is the vector of skewness,  $p_2$  is the scale parameter,  $\mu = 0$  is the location parameter and  $\tau = 1$  the kurtosis parameter.

## Value

A vector containing the user selected density values at the user specified points  $s$ .

## Author(s)

Dimitrios Bagkavos and Lucia Gamez Gallardo

R implementation and documentation: Dimitrios Bagkavos <dimitrios.bagkavos@gmail.com>, Lucia Gamez Gallardo <gamezgallardolucia@gmail.com>

## References

Bagkavos D., Patil P.N., Wood A.T.A. (2016), A Numerical Study of the Power Function of a New Symmetry Test. In: Cao R., Gonzalez Manteiga W., Romo J. (eds) Nonparametric Statistics. Springer Proceedings in Mathematics and Statistics, vol 175, Springer.

## See Also

[r.sample](#), [q.sample](#), [p.sample](#)

## Examples

```
selected.dens <- "weib" #select Weibull
shape <- 2 # specify shape parameter
scale <- 1 # specify scale parameter
xout <- seq(0.1,5,length=50) #design point
pdfsqcdf(xout,selected.dens,shape,scale) # calculate pdfsqcdf function at xout
```

---

|              |                                |
|--------------|--------------------------------|
| pdfsqcdfstar | Calculate $f^2(x)(1 - F(x))$ . |
|--------------|--------------------------------|

---

### Description

Return the product  $f^2(x)(1 - F(x))$ .

### Usage

```
pdfsqcdfstar(s,dist, p1,p2)
```

### Arguments

|      |                                                                                                      |
|------|------------------------------------------------------------------------------------------------------|
| s    | A scalar or vector: the x-axis grid points where the probability density function will be evaluated. |
| dist | Character string, used as a switch to the user selected distribution function (see details below).   |
| p1   | A scalar. Parameter 1 (vector or object) of the selected density.                                    |
| p2   | A scalar. Parameter 2 (vector or object) of the selected density.                                    |

### Details

Based on user-specified argument `dist`, the function returns the value of  $f^2(x)(1 - F(x))dx$ , used in the definitions of  $\rho_p^*$ ,  $\rho_p$  and their exact versions.

Supported distributions (along with the corresponding `dist` values) are:

- weib: The weibull distribution is implemented as

$$f(s; p_1, p_2) = \frac{p_1}{p_2} \left( \frac{s}{p_2} \right)^{p_1-1} \exp \left\{ - \left( \frac{s}{p_2} \right)^{p_1} \right\}$$

with  $s \geq 0$  where  $p_1$  is the shape parameter and  $p_2$  the scale parameter.

- lognorm: The lognormal distribution is implemented as

$$f(s) = \frac{1}{p_2 s \sqrt{2\pi}} e^{-\frac{(\log s - p_1)^2}{2p_2^2}}$$

where  $p_1$  is the mean and  $p_2$  is the standard deviation of the distribution.

- norm: The normal distribution is implemented as

$$f(s) = \frac{1}{p_2 \sqrt{2\pi}} e^{-\frac{(s - p_1)^2}{2p_2^2}}$$

where  $p_1$  is the mean and the  $p_2$  is the standard deviation of the distribution.

- uni: The uniform distribution is implemented as

$$f(s) = \frac{1}{p_2 - p_1}$$

for  $p_1 \leq s \leq p_2$ .

- cauchy: The cauchy distribution is implemented as

$$f(s) = \frac{1}{\pi p_2 \left\{ 1 + \left( \frac{s-p_1}{p_2} \right)^2 \right\}}$$

where  $p_1$  is the location parameter and  $p_2$  the scale parameter.

- fnorm: The half normal distribution is implemented as

$$2f(s) - 1$$

where

$$f(s) = \frac{1}{sd\sqrt{2\pi}} e^{-\frac{s^2}{2sd^2}},$$

and  $sd = \sqrt{\pi/2}/p_1$ .

- normmixt: The normal mixture distribution is implemented as

$$f(s) = p_1 \frac{1}{p_2[2]\sqrt{2\pi}} e^{-\frac{(s-p_2[1])^2}{2p_2[2]^2}} + (1-p_1) \frac{1}{p_2[4]\sqrt{2\pi}} e^{-\frac{(s-p_2[3])^2}{2p_2[4]^2}}$$

where  $p_1$  is a mixture component (scalar) and  $p_2$  a vector of parameters for the mean and variance of the two mixture components  $p_2 = c(mean1, sd1, mean2, sd2)$ .

- skewnorm: The skew normal distribution with parameter  $p_1$  is implemented as

$$f(s) = 2\phi(s)\Phi(p_1 s)$$

.

- fas: The Fernandez and Steel distribution is implemented as

$$f(s; p_1, p_2) = \frac{2}{p_1 + \frac{1}{p_1}} \left\{ f_t(s/p_1; p_2) I_{\{s \geq 0\}} + f_t(p_1 s; p_2) I_{\{s < 0\}} \right\}$$

where  $f_t(x; \nu)$  is the p.d.f. of the  $t$  distribution with  $\nu = 5$  degrees of freedom.  $p_1$  controls the skewness of the distribution with values between  $(0, +\infty)$  and  $p_2$  denotes the degrees of freedom.

- shash: The Sinh-Arcsinh distribution is implemented as

$$f(s; \mu, p_1, p_2, \tau) = \frac{ce^{-r^2/2}}{\sqrt{2\pi}} \frac{1}{p_2} \frac{1}{2} \sqrt{1+z^2}$$

where  $r = \sinh(\sinh(z) - (-p_1))$ ,  $c = \cosh(\sinh(z) - (-p_1))$  and  $z = ((s - \mu)/p_2)$ .  $p_1$  is the vector of skewness,  $p_2$  is the scale parameter,  $\mu = 0$  is the location parameter and  $\tau = 1$  the kurtosis parameter.

**Value**

A vector containing the user selected density values at the user specified points  $s$ .

**Author(s)**

Dimitrios Bagkavos and Lucia Gamez Gallardo

R implementation and documentation: Dimitrios Bagkavos <dimitrios.bagkavos@gmail.com>, Lucia Gamez Gallardo <gamezgallardolucia@gmail.com>

**References**

Bagkavos D., Patil P.N., Wood A.T.A. (2016), A Numerical Study of the Power Function of a New Symmetry Test. In: Cao R., Gonzalez Manteiga W., Romo J. (eds) Nonparametric Statistics. Springer Proceedings in Mathematics and Statistics, vol 175, Springer.

**See Also**

[r.sample](#), [q.sample](#), [p.sample](#)

**Examples**

```
selected.dens <- "weib" #select Weibull
shape <- 2 # specify shape parameter
scale <- 1 # specify scale parameter
xout <- seq(0.1,5,length=50) #design point
pdfsqcdfstar(xout,selected.dens,shape,scale) #return  $f^2(x)F(x)$ 
```

---

pdfthird

---

*Calculate  $f^3(x)$* 


---

**Description**

Return the value of  $f^3(x)$ .

**Usage**

```
pdfthird(s,dist, p1,p2)
```

**Arguments**

|        |                                                                                                      |
|--------|------------------------------------------------------------------------------------------------------|
| $s$    | A scalar or vector: the x-axis grid points where the probability density function will be evaluated. |
| $dist$ | Character string, used as a switch to the user selected distribution function (see details below).   |
| $p1$   | A scalar. Parameter 1 (vector or object) of the selected density.                                    |
| $p2$   | A scalar. Parameter 2 (vector or object) of the selected density.                                    |

### Details

Based on user-specified argument `dist`, the function returns the value of  $f^3(x)dx$ , used in the definitions of  $\rho_p^*$ ,  $\rho_p$  and their exact versions.

Supported distributions (along with the corresponding `dist` values) are:

- `weib`: The weibull distribution is implemented as

$$f(s; p_1, p_2) = \frac{p_1}{p_2} \left( \frac{s}{p_2} \right)^{p_1-1} \exp \left\{ - \left( \frac{s}{p_2} \right)^{p_1} \right\}$$

with  $s \geq 0$  where  $p_1$  is the shape parameter and  $p_2$  the scale parameter.

- `lognorm`: The lognormal distribution is implemented as

$$f(s) = \frac{1}{p_2 s \sqrt{2\pi}} e^{-\frac{(\log s - p_1)^2}{2p_2^2}}$$

where  $p_1$  is the mean and  $p_2$  is the standard deviation of the distribution.

- `norm`: The normal distribution is implemented as

$$f(s) = \frac{1}{p_2 \sqrt{2\pi}} e^{-\frac{(s-p_1)^2}{2p_2^2}}$$

where  $p_1$  is the mean and the  $p_2$  is the standard deviation of the distribution.

- `uni`: The uniform distribution is implemented as

$$f(s) = \frac{1}{p_2 - p_1}$$

for  $p_1 \leq s \leq p_2$ .

- `cauchy`: The cauchy distribution is implemented as

$$f(s) = \frac{1}{\pi p_2 \left\{ 1 + \left( \frac{s-p_1}{p_2} \right)^2 \right\}}$$

where  $p_1$  is the location parameter and  $p_2$  the scale parameter.

- `fnorm`: The half normal distribution is implemented as

$$2f(s) - 1$$

where

$$f(s) = \frac{1}{sd\sqrt{2\pi}} e^{-\frac{s^2}{2sd^2}},$$

and  $sd = \sqrt{\pi/2}/p_1$ .

- `normmixt`: The normal mixture distribution is implemented as

$$f(s) = p_1 \frac{1}{p_2[2]\sqrt{2\pi}} e^{-\frac{(s-p_2[1])^2}{2p_2[2]^2}} + (1-p_1) \frac{1}{p_2[4]\sqrt{2\pi}} e^{-\frac{(s-p_2[3])^2}{2p_2[4]^2}}$$

where  $p_1$  is a mixture component (scalar) and  $p_2$  a vector of parameters for the mean and variance of the two mixture components  $p_2 = c(mean1, sd1, mean2, sd2)$ .

- skewnorm: The skew normal distribution with parameter  $p_1$  is implemented as

$$f(s) = 2\phi(s)\Phi(p_1 s)$$

- fas: The Fernandez and Steel distribution is implemented as

$$f(s; p_1, p_2) = \frac{2}{p_1 + \frac{1}{p_1}} \{f_t(s/p_1; p_2)I_{\{s \geq 0\}} + f_t(p_1 s; p_2)I_{\{s < 0\}}\}$$

where  $f_t(x; \nu)$  is the p.d.f. of the  $t$  distribution with  $\nu = 5$  degrees of freedom.  $p_1$  controls the skewness of the distribution with values between  $(0, +\infty)$  and  $p_2$  denotes the degrees of freedom.

- shash: The Sinh-Arcsinh distribution is implemented as

$$f(s; \mu, p_1, p_2, \tau) = \frac{ce^{-r^2/2}}{\sqrt{2\pi}} \frac{1}{p_2} \frac{1}{2} \sqrt{1+z^2}$$

where  $r = \sinh(\sinh(z) - (-p_1))$ ,  $c = \cosh(\sinh(z) - (-p_1))$  and  $z = ((s - \mu)/p_2)$ .  $p_1$  is the vector of skewness,  $p_2$  is the scale parameter,  $\mu = 0$  is the location parameter and  $\tau = 1$  the kurtosis parameter.

## Value

A vector containing the user selected density values at the user specified points  $s$ .

## Author(s)

Dimitrios Bagkavos and Lucia Gamez Gallardo

R implementation and documentation: Dimitrios Bagkavos <dimitrios.bagkavos@gmail.com>, Lucia Gamez Gallardo <gamezgallardolucia@gmail.com>

## References

Bagkavos D., Patil P.N., Wood A.T.A. (2016), A Numerical Study of the Power Function of a New Symmetry Test. In: Cao R., Gonzalez Manteiga W., Romo J. (eds) Nonparametric Statistics. Springer Proceedings in Mathematics and Statistics, vol 175, Springer.

## See Also

[pdfsq](#), [pdfsqcdf](#), [pdfsqcdfstar](#)

## Examples

```
selected.dens <- "weib" #select Weibull
shape <- 2 # specify shape parameter
scale <- 1 # specify scale parameter
xout <- seq(0.1,5,length=50) #design point
pdfthird(xout,selected.dens,shape,scale) # calculate density to the cube at xout
```

---

q.sample

Switch between a range of available quantile functions.

---

### Description

Returns the quantiles of selected distributions at user specified locations.

### Usage

```
q.sample(s,dist, p1=0,p2=1)
```

### Arguments

|      |                                                                                                    |
|------|----------------------------------------------------------------------------------------------------|
| s    | A scalar or vector: the probabilities where the quantile function will be evaluated.               |
| dist | Character string, used as a switch to the user selected distribution function (see details below). |
| p1   | A scalar. Parameter 1 (vector or object) of the selected distribution.                             |
| p2   | A scalar. Parameter 2 (vector or object) of the selected distribution.                             |

### Details

Based on user-specified argument `dist`, the function returns the value of the quantile function at `s`. Supported distributions (along with the corresponding `dist` values) are:

- `weib`: The quantile function for the weibull distribution is implemented as

$$Q(s) = p_1(-\log(1-s))^{1/p_2}$$

where  $p_1$  is the shape parameter and  $p_2$  the scale parameter.

- `lognorm`: The lognormal distribution has quantile function implemented as

$$Q(s) = \exp \left\{ p_1 + \sqrt{2p_2^2} \operatorname{erf}^{-1}(2s-1) \right\}$$

where  $p_1$  is the mean,  $p_2$  is the standard deviation and `erf` is the Gauss error function.

- `norm`: The normal distribution has quantile function implemented as

$$Q(p) = \Phi^{-1}(s; p_1, p_2)$$

where  $p_1$  is the mean and the  $p_2$  is the standard deviation.

- `uni`: The uniform distribution has quantile function implemented as

$$Q(s; p_1, p_2) = s(p_2 - p_1) + p_1$$

for  $p_1 < s < p_2$ .

- **cauchy**: The cauchy distribution has quantile function implemented as

$$Q(s) = p_1 + p_2 \tan \left\{ \pi \left( s - \frac{1}{2} \right) \right\}$$

where  $p_1$  is the location parameter and  $p_2$  the scale parameter.

- **fnorm**: The half normal distribution has quantile function implemented as

$$Q(s) = p_1 \sqrt{2} \text{erf}^{-1}(s)$$

where  $p_1$  is the standard deviation of the distribution.

- **normmix**: The quantile function normal mixture distribution is estimated numerically, based on the built in quantile function.
- **skewnorm**: There is no closed form expression for the quantile function of the skew normal distribution. For this reason, the quantiles are calculated through the qsn function of the sn package.
- **fas**: There is no closed form expression for the quantile function of the Fernandez and Steel distribution. For this reason, the quantiles are calculated through the qskt function of the skewt package.
- **shash**: There is no closed form expression for the quantile function of the Sinh-Arcsinh distribution. For this reason, the quantiles are calculated through the qSHASHo function of the gamlss package.

### Value

A vector containing the quantile values at the user specified points  $s$ .

### Author(s)

Dimitrios Bagkavos and Lucia Gamez Gallardo

R implementation and documentation: Dimitrios Bagkavos <dimitrios.bagkavos@gmail.com> ,  
Lucia Gamez Gallardo <gamezgallardolucia@gmail.com>

### References

Bagkavos D., Patil P.N., Wood A.T.A. (2016), A Numerical Study of the Power Function of a New Symmetry Test. In: Cao R., Gonzalez Manteiga W., Romo J. (eds) Nonparametric Statistics. Springer Proceedings in Mathematics and Statistics, vol 175, Springer.

### See Also

[r.sample](#), [d.sample](#), [p.sample](#)

### Examples

```
selected.q <- "norm" #select Normal as the distribution
shape <- 2 # specify shape parameter
scale <- 2 # specify scale parameter
xout <- seq(0.1,1,length=50) #design point where the quantile function is evaluated
q.sample(xout,selected.q,shape,scale) # calculate quantiles at xout
```

---

r.sample

Switch between a range of available random number generators.

---

### Description

Generate a random sample of size  $n$  out of a range of available distributions.

### Usage

```
r.sample(s, dist, p1=0, p2=1)
```

### Arguments

- |      |                                                                                                    |
|------|----------------------------------------------------------------------------------------------------|
| s    | A scalar which specifies the size of the random sample drawn.                                      |
| dist | Character string, used as a switch to the user selected distribution function (see details below). |
| p1   | A scalar. Parameter 1 (vector or object) of the selected distribution.                             |
| p2   | A scalar. Parameter 2 (vector or object) of the selected distribution.                             |

### Details

Based on user-specified argument `dist`, the function returns a random sample of size  $s$  from the corresponding distribution.

Supported distributions (along with the corresponding `dist` values) are:

- `weib`: The weibull distribution is implemented as

$$f(s; p_1, p_2) = \frac{p_1}{p_2} \left( \frac{s}{p_2} \right)^{p_1-1} \exp \left\{ - \left( \frac{s}{p_2} \right)^{p_1} \right\}$$

with  $s \geq 0$  where  $p_1$  is the shape parameter and  $p_2$  the scale parameter.

- `lognorm`: The lognormal distribution is implemented as

$$f(s) = \frac{1}{p_2 s \sqrt{2\pi}} e^{-\frac{(\log s - p_1)^2}{2p_2^2}}$$

where  $p_1$  is the mean and  $p_2$  is the standard deviation of the distribution.

- `norm`: The normal distribution is implemented as

$$f(s) = \frac{1}{p_2 \sqrt{2\pi}} e^{-\frac{(s - p_1)^2}{2p_2^2}}$$

where  $p_1$  is the mean and the  $p_2$  is the standard deviation of the distribution.

- `uni`: The uniform distribution is implemented as

$$f(s) = \frac{1}{p_2 - p_1}$$

for  $p_1 \leq s \leq p_2$ .

- **cauchy**: The cauchy distribution is implemented as

$$f(s) = \frac{1}{\pi p_2 \left\{ 1 + \left( \frac{s-p_1}{p_2} \right)^2 \right\}}$$

where  $p_1$  is the location parameter and  $p_2$  the scale parameter.

- **fnorm**: The half normal distribution is implemented as

$$2f(s) - 1$$

where

$$f(s) = \frac{1}{sd\sqrt{2\pi}} e^{-\frac{s^2}{2sd^2}},$$

and  $sd = \sqrt{\pi/2}/p_1$ .

- **normmixt**: The normal mixture distribution is implemented as

$$f(s) = p_1 \frac{1}{p_2[2]\sqrt{2\pi}} e^{-\frac{(s-p_2[1])^2}{2p_2[2]^2}} + (1-p_1) \frac{1}{p_2[4]\sqrt{2\pi}} e^{-\frac{(s-p_2[3])^2}{2p_2[4]^2}}$$

where  $p_1$  is a mixture component (scalar) and  $p_2$  a vector of parameters for the mean and variance of the two mixture components  $p_2 = c(mean1, sd1, mean2, sd2)$ .

- **skewnorm**: The skew normal distribution with parameter  $p_1$  is implemented as

$$f(s) = 2\phi(s)\Phi(p_1 s)$$

- **fas**: The Fernandez and Steel distribution is implemented as

$$f(s; p_1, p_2) = \frac{2}{p_1 + \frac{1}{p_1}} \left\{ f_t(s/p_1; p_2) I_{\{s \geq 0\}} + f_t(p_1 s; p_2) I_{\{s < 0\}} \right\}$$

where  $f_t(x; \nu)$  is the p.d.f. of the  $t$  distribution with  $\nu = 5$  degrees of freedom.  $p_1$  controls the skewness of the distribution with values between  $(0, +\infty)$  and  $p_2$  denotes the degrees of freedom.

- **shash**: The Sinh-Arcsinh distribution is implemented as

$$f(s; \mu, p_1, p_2, \tau) = \frac{ce^{-r^2/2}}{\sqrt{2\pi}} \frac{1}{p_2} \frac{1}{2} \sqrt{1+z^2}$$

where  $r = \sinh(\sinh(z) - (-p_1))$ ,  $c = \cosh(\sinh(z) - (-p_1))$  and  $z = ((s - \mu)/p_2)$ .  $p_1$  is the vector of skewness,  $p_2$  is the scale parameter,  $\mu = 0$  is the location parameter and  $\tau = 1$  the kurtosis parameter.

## Value

A vector of random values at the user specified points  $s$ .

**Author(s)**

Dimitrios Bagkavos and Lucia Gamez Gallardo

R implementation and documentation: Dimitrios Bagkavos <dimitrios.bagkavos@gmail.com> ,  
Lucia Gamez Gallardo <gamezgallardolucia@gmail.com>

**References**

Bagkavos D., Patil P.N., Wood A.T.A. (2016), A Numerical Study of the Power Function of a New Symmetry Test. In: Cao R., Gonzalez Manteiga W., Romo J. (eds) Nonparametric Statistics. Springer Proceedings in Mathematics and Statistics, vol 175, Springer.

**See Also**

[d.sample](#), [q.sample](#), [p.sample](#)

**Examples**

```
selected.r <- "norm" #select Normal as the distribution
shape <- 2 # specify shape parameter
scale <- 1 # specify scale parameter
n <- 100 # specify sample size
r.sample(n,selected.r,shape,scale) # calculate CDF at the designated point
```

---

|       |                                                                                                                         |
|-------|-------------------------------------------------------------------------------------------------------------------------|
| Rho.p | <i>Calculates <math>\rho_p</math>, used in the implementation of the strong asymmetry measure <math>\eta(X)</math>.</i> |
|-------|-------------------------------------------------------------------------------------------------------------------------|

---

**Description**

Estimates  $\rho_p$ , used in the calculation of the strong asymmetry measure  $\eta(X)$ .

**Usage**

```
Rho.p(xin, p.param, dist, p1=0, p2=1)
```

**Arguments**

|         |                                                                          |
|---------|--------------------------------------------------------------------------|
| xin     | A vector of data points - the available sample.                          |
| p.param | A parameter with the value greater than or equal to 1/2 and less than 1. |
| dist    | Character string, specifies selected distribution function.              |
| p1      | A scalar. Parameter 1 (vector or object) of the selected distribution.   |
| p2      | A scalar. Parameter 2 (vector or object) of the selected distribution.   |

## Details

Implements the quantity:

$$\frac{2\sqrt{3} - \int_{-\infty}^{\xi_p} f^2(x)F(x) dx - \frac{p}{2} \int_{-\infty}^{\xi_p} f^2(x) dx}{p \left\{ p \int_{-\infty}^{\xi_p} f^3(x) dx - \left( \int_{-\infty}^{\xi_p} f^2(x) dx \right)^2 \right\}^{1/2}}$$

defined on page 6 [Patil, Bagkavos and Wood](#), see also (4) in [Bagkavos, Patil and Wood](#). Estimation of the p.d.f. and c.d.f. functions is currently performed by maximum likelihood as e.g. kernel estimates inherit large amount of variance to  $\rho_p$ .

## Value

Returns a scalar, the value of  $\rho_p$ .

## Author(s)

Dimitrios Bagkavos and Lucia Gamez Gallardo

R implementation and documentation: Dimitrios Bagkavos <dimitrios.bagkavos@gmail.com> ,  
Lucia Gamez Gallardo <gamezgallardolucia@gmail.com>

## References

- [Patil P.N., Bagkavos D. and Wood A.T.A., \(2014\). A measure of asymmetry based on a new necessary and sufficient condition for symmetry, Sankhya A, 76, 123–145.](#)
- [Bagkavos D., Patil P.N., Wood A.T.A. \(2016\), A Numerical Study of the Power Function of a New Symmetry Test. In: Cao R., González Manteiga W., Romo J. \(eds\) Nonparametric Statistics. Springer Proceedings in Mathematics and Statistics, vol 175, Springer.](#)

## See Also

[Rho.p.exact](#), [Rhostar.p](#), [Rhostar.p.exact](#)

## Examples

```
set.seed(1234)

selected.r <- "weib" #select Weibull as the distribution
shape <- 1 # specify shape parameter
scale <- 1 # specify scale parameter
n <- 100 # specify sample size
param <- 0.9 # specify parameter
xout<-r.sample(n,selected.r,shape,scale) # specify sample
Rho.p(xout,param,selected.r,shape,scale) # calculate Rho.p
#-0.06665222 # returns the result

selected.r2 <- "norm" #select Normal as the distribution
n <- 100 # specify sample size
mean <- 0 # specify the mean
sd <- 1 # specify the variance
```

```

param <- 0.9 # specify parameter
xout <- r.sample(n,selected.r2,mean,sd) # specify sample
Rho.p(xout,param,selected.r2,mean,sd) # calculate Rho.p
#-0.1005591 # returns the result

selected.r3 <- "cauchy" #select Cauchy as the distribution
n <- 100 # specify sample size
location <- 0 # specify the location parameter
scale <- 1 # specify the scale parameter
param <- 0.9 # specify parameter
xout<-r.sample(n,selected.r3,location,scale) # specify sample
Rho.p(xout,param,selected.r3,location,scale) # calculate Rho.p
#-0.0580943 # returns the result

```

---

Rho.p.exact

*Calculates the exact value  $\rho_p$ , used in the implementation of the strong asymmetry measure  $\eta(X)$ .*

---

### Description

Returns  $\rho_p$ , used in the calculation of the strong asymmetry measure  $\eta(X)$ .

### Usage

```
Rho.p.exact(xin, p.param, dist, p1=0, p2=1)
```

### Arguments

|         |                                                                          |
|---------|--------------------------------------------------------------------------|
| xin     | A vector of data points - the available sample.                          |
| p.param | A parameter with the value greater than or equal to 1/2 and less than 1. |
| dist    | Character string, specifies selected distribution function.              |
| p1      | A scalar. Parameter 1 (vector or object) of the selected distribution.   |
| p2      | A scalar. Parameter 2 (vector or object) of the selected distribution.   |

### Details

Implements the quantity:

$$\frac{2\sqrt{3} - \int_{-\infty}^{\xi_p} f^2(x)F(x) dx - \frac{p}{2} \int_{-\infty}^{\xi_p} f^2(x) dx}{p \left\{ p \int_{-\infty}^{\xi_p} f^3(x) dx - \left( \int_{-\infty}^{\xi_p} f^2(x) dx \right)^2 \right\}^{1/2}}$$

defined on page 6 [Patil, Bagkavos and Wood](#), see also (4) in [Bagkavos, Patil and Wood](#) . This implementation uses exact calculation of the functionals in the definition of  $\rho_p$ .

**Value**

Returns a scalar, the exact value of  $\rho_p$ .

**Author(s)**

Dimitrios Bagkavos and Lucia Gamez Gallardo

R implementation and documentation: Dimitrios Bagkavos <dimitrios.bagkavos@gmail.com> ,  
Lucia Gamez Gallardo <gamezgallardolucia@gmail.com>

**References**

- Patil P.N., Bagkavos D. and Wood A.T.A., (2014). A measure of asymmetry based on a new necessary and sufficient condition for symmetry, *Sankhya A*, 76, 123–145.
- Bagkavos D., Patil P.N., Wood A.T.A. (2016), A Numerical Study of the Power Function of a New Symmetry Test. In: Cao R., Gonzalez Manteiga W., Romo J. (eds) *Nonparametric Statistics. Springer Proceedings in Mathematics and Statistics*, vol 175, Springer.

**See Also**

[Rho.p](#), [Rhostar.p](#), [Rhostar.p.exact](#)

**Examples**

```
set.seed(1234)

selected.r <- "weib" #select Weibull as the distribution
shape <- 1 # specify shape parameter
scale <- 1 # specify scale parameter
n <- 100 # specify sample size
param <- 0.9 # specify parameter
xout<-r.sample(n,selected.r,shape,scale) # specify sample
Rho.p.exact(xout,param,selected.r,shape,scale) # calculate Rho.p.exact
#-0.06665222 # returns the result

selected.r2 <- "norm" #select Normal as the distribution
n <- 100 # specify sample size
mean <- 0 # specify the mean
sd <- 1 # specify the variance
param <- 0.9 # specify parameter
xout <-r.sample(n,selected.r2,mean,sd) # specify sample
Rho.p.exact(xout,param,selected.r2,mean,sd) # calculate Rho.p.exact
#-0.2384271 # returns the result

selected.r3 <- "cauchy" #select Cauchy as the distribution
n <- 100 # specify sample size
location <- 0 # specify the location parameter
scale <- 1 # specify the scale parameter
param <- 0.9 # specify parameter
xout<-r.sample(n,selected.r3,location,scale) # specify sample
```

```
Rho.p.exact(xout,param,selected.r3,location,scale) # calculate Rho.p.exact
#-0.02340374 # returns the result
```

---

|           |                                                                                                                           |
|-----------|---------------------------------------------------------------------------------------------------------------------------|
| Rhostar.p | <i>Calculates <math>\rho_p^*</math>, used in the implementation of the strong asymmetry measure <math>\eta(X)</math>.</i> |
|-----------|---------------------------------------------------------------------------------------------------------------------------|

---

### Description

Estimates  $\rho_p^*$ , used in the calculation of the strong asymmetry measure  $\eta(X)$ .

### Usage

```
Rhostar.p(xin, p.param, dist, p1, p2)
```

### Arguments

|         |                                                                          |
|---------|--------------------------------------------------------------------------|
| xin     | A vector of data points - the available sample.                          |
| p.param | A parameter with the value greater than or equal to 1/2 and less than 1. |
| dist    | Character string, specifies selected distribution function.              |
| p1      | A scalar. Parameter 1 (vector or object) of the selected distribution.   |
| p2      | A scalar. Parameter 2 (vector or object) of the selected distribution.   |

### Details

Implements the quantity

$$\frac{2\sqrt{3} - \int_{\xi_{1-p}}^{\infty} f^2(x)(1 - F(x)) dx + \frac{p}{2} \int_{\xi_{1-p}}^{\infty} f^2(x) dx}{p \left\{ p \int_{\xi_{1-p}}^{\infty} f^3(x) dx - \left( \int_{\xi_{1-p}}^{\infty} f^2(x) dx \right)^2 \right\}^{1/2}}$$

defined on page 6 [Patil, Bagkavos and Wood \(2014\)](#), see also (5) in [Bagkavos, Patil and Wood \(2016\)](#). Estimation of the p.d.f. and c.d.f. functions is currently performed by maximum likelihood as e.g. kernel estimates inherit large amount of variance to  $\rho_p^*$ .

### Value

Returns a scalar, the value of  $\rho_p^*$ .

### Author(s)

Dimitrios Bagkavos and Lucia Gamez Gallardo

R implementation and documentation: Dimitrios Bagkavos <dimitrios.bagkavos@gmail.com>, Lucia Gamez Gallardo <gamezgallardolucia@gmail.com>

## References

- Patil P.N., Bagkavos D. and Wood A.T.A., (2014). A measure of asymmetry based on a new necessary and sufficient condition for symmetry, *Sankhya A*, 76, 123–145.
- Bagkavos D., Patil P.N., Wood A.T.A. (2016), A Numerical Study of the Power Function of a New Symmetry Test. In: Cao R., Gonzalez Manteiga W., Romo J. (eds) *Nonparametric Statistics. Springer Proceedings in Mathematics and Statistics*, vol 175, Springer.

## See Also

[Rho.p](#), [Rhostar.p.exact](#), [Rho.p.exact](#)

## Examples

```
set.seed(1234)

selected.r <- "weib" #select Weibull as the distribution
shape <- 1 # specify shape parameter
scale <- 1 # specify scale parameter
n <- 100 # specify sample size
param <- 0.9 # specify parameter
xout<-r.sample(n,selected.r,shape,scale) # specify sample
Rhostar.p(xout,param,selected.r,shape,scale) # calculate Rhostar.p
#-0.08936363 # returns the result

selected.r2 <- "norm" #select Normal as the distribution
n <- 100 # specify sample size
mean <- 0 # specify the mean
sd <- 1 # specify the variance
param <- 0.9 # specify parameter
xout <-r.sample(n,selected.r2,mean,sd) # specify sample
Rhostar.p(xout,param,selected.r2,mean,sd) # calculate Rhostar.p
#-0.02302223 # returns the result

selected.r3 <- "cauchy" #select Cauchy as the distribution
n <- 100 # specify sample size
location <- 0 # specify the location parameter
scale <- 1 # specify the scale parameter
param <- 0.9 # specify parameter
xout<-r.sample(n,selected.r3,location,scale) # specify sample
Rhostar.p(xout,param,selected.r3,location,scale) # calculate Rhostar.p
#0.02043852 # returns the result
```

---

Rhostar.p.exact

*Calculates the exact value of  $p\_p^*$ , used in the implementation of the strong asymmetry measure  $\eta(X)$ .*

---

**Description**

Returns  $\rho_p^*$ , used in the calculation of the strong asymmetry measure  $\eta(X)$ .

**Usage**

```
Rhostar.p.exact(xin, p.param, dist, p1, p2)
```

**Arguments**

|         |                                                                          |
|---------|--------------------------------------------------------------------------|
| xin     | A vector of data points - the available sample.                          |
| p.param | A parameter with the value greater than or equal to 1/2 and less than 1. |
| dist    | Character string, specifies selected distribution function.              |
| p1      | A scalar. Parameter 1 (vector or object) of the selected distribution.   |
| p2      | A scalar. Parameter 2 (vector or object) of the selected distribution.   |

**Details**

Implements the quantity

$$\frac{2\sqrt{3} - \int_{\xi_{1-p}}^{\infty} f^2(x)(1 - F(x)) dx + \frac{p}{2} \int_{\xi_{1-p}}^{\infty} f^2(x) dx}{p \left\{ p \int_{\xi_{1-p}}^{\infty} f^3(x) dx - \left( \int_{\xi_{1-p}}^{\infty} f^2(x) dx \right)^2 \right\}^{1/2}}$$

defined on page 6 [Patil, Bagkavos and Wood \(2014\)](#), see also (5) in [Bagkavos, Patil and Wood \(2016\)](#). This implementation uses exact calculation of the functionals in the definition of  $\rho_p^*$ .

**Value**

Returns a scalar, the exact value of  $\rho_p^*$ .

**Author(s)**

Dimitrios Bagkavos and Lucia Gamez Gallardo

R implementation and documentation: Dimitrios Bagkavos <dimitrios.bagkavos@gmail.com> ,  
Lucia Gamez Gallardo <gamezgallardolucia@gmail.com>

**References**

- [Patil P.N., Bagkavos D. and Wood A.T.A., \(2014\). A measure of asymmetry based on a new necessary and sufficient condition for symmetry, Sankhya A, 76, 123–145.](#)
- [Bagkavos D., Patil P.N., Wood A.T.A. \(2016\), A Numerical Study of the Power Function of a New Symmetry Test. In: Cao R., Gonzalez Manteiga W., Romo J. \(eds\), Nonparametric Statistics. Springer Proceedings in Mathematics and Statistics, vol 175, Springer.](#)

**See Also**

[Rho.p](#), [Rhostar.p](#), [Rho.p.exact](#)

**Examples**

```

set.seed(1234)

selected.r <- "weib" #select Weibull as the distribution
shape <- 1 # specify shape parameter
scale <- 1 # specify scale parameter
n <- 100 # specify sample size
param <- 0.9 # specify parameter
xout<-r.sample(n,selected.r,shape,scale) # specify sample
Rhostar.p.exact(xout,param,selected.r,shape,scale) # calculate Rhostar.p.exact
#-0.05206678 # returns the result

selected.r2 <- "norm" #select Normal as the distribution
n <- 100 # specify sample size
mean <- 0 # specify the mean
sd <- 1 # specify the variance
param <- 0.9 # specify parameter
xout <-r.sample(n,selected.r2,mean,sd) # specify sample
Rhostar.p.exact(xout,param,selected.r2,mean,sd) # calculate Rhostar.p.exact
#-0.008687447 # returns the result

selected.r3 <- "cauchy" #select Cauchy as the distribution
n <- 100 # specify sample size
location <- 0 # specify the location parameter
scale <- 1 # specify the scale parameter
param <- 0.9 # specify parameter
xout<-r.sample(n,selected.r3,location,scale) # specify sample
Rhostar.p.exact(xout,param,selected.r3,location,scale) # calculate Rhostar.p.exact
#0.0280602 # returns the result

```

---

SimpsonInt

*Simpson integration*


---

**Description**

Implements simpson's extended integration rule.

**Usage**

```
SimpsonInt(xin,h)
```

**Arguments**

|     |                                                                   |
|-----|-------------------------------------------------------------------|
| xin | A vector of design points where the integral will be evaluated.   |
| h   | Assuming $a < b$ and $n$ is a positive integer. $h = (b - a)/n$ . |

**Details**

Simpson's extended numerical integration rule is implemented for  $n+1$  equally spaced subdivisions (where  $n$  is even) of  $[a, b]$  as

$$\int_a^b f(x) dx = \frac{h}{3} \{f(a) + 4f(x_1) + 2f(x_2) + 4f(x_3) + 2f(x_4) + \dots + 4f(x_{n-1}) + f(b)\}$$

where  $hx = (b - a)/n$  and  $x_i = a + ihx$ . Simpson's rule will return an exact result when the polynomial in question has a degree of three or less. For other functions, Simpson's Rule only gives an approximation.

**Value**

A scalar, the approximate value of the integral.

**Author(s)**

Dimitrios Bagkavos and Lucia Gamez Gallardo

R implementation and documentation: Dimitrios Bagkavos <dimitrios.bagkavos@gmail.com> ,  
Lucia Gamez Gallardo <gamezgallardolucia@gmail.com>

**References**

[Simpson's Rule](#)

**Examples**

```
x.in<- seq(0,pi/4,length=5)
h.out <- pi/8
SimpsonInt(x.in,h.out)
```

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