

Package ‘LinkedGASP’

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Description Prototypes for construction of a Gaussian Stochastic Process emulator (GASP) of a computer model. This is done within the objective Bayesian implementation of the GASP. The package allows for construction of a linked GASP of the composite computer model. Computational implementation follows the mathematical exposition given in publication: Ksenia N. Kzyurova, James O. Berger, Robert L. Wolpert. Coupling computer models through linking their statistical emulators. SIAM/ASA Journal on Uncertainty Quantification, 6(3): 1151-1171, (2018).<[DOI:10.1137/17M1157702](https://doi.org/10.1137/17M1157702)>.

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emp_GASP_plot	<i>Empirical linked GASP plot</i>
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Description

Function plots the empirical true linked emulator in case of one-dimensional input.

Usage

```
emp_GASP_plot(em, fun, data, emul_type, exp.ql, exp.qu, labels, ylab, xlab, ylim,
  col_CI_area, col_points, col_fun, col_mean, points)
```

Arguments

em	the returned output from the function eval_type1_GASP(...) or eval_type2_GASP(...).
fun	Simulator function. Currently only one-dimensional input is supported.
data	Training data and smoothness. The same as supplied to eval_GASP_RFP(...) for construction of the GASP.
emul_type	A text string which provides description of an emulator.
exp.ql	Quantile 0.025
exp.qu	Quantile 0.975
labels	As in standard R plot.
ylab	As in standard R plot.
xlab	As in standard R plot.
ylim	As in standard R plot.
col_CI_area	Color of a credible area.
col_points	Color of the training points.
col_fun	Color of a simulator function.
col_mean	Color of the emulator of the GASP mean.
points	Default is FALSE. To plot or not the training points.

Value

Plot

Author(s)

Ksenia N. Kzyurova, kseniak.ucoz.net

Examples

```

## Function f1 is a simulator
f1<-function(x){sin(pi*x)}
## Function f2 is a simulator
f2<-function(x){cos(5*x)}

## Function f2(f1) is a simulator of a composite model
f2f1 <- function(x){f2(f1(x))}

## One-dimensional inputs are x1
x1 <- seq(-1,1,.37)

## The following contains the list of data inputs (training) and outputs (fD) together with the
## assumed fixed smoothness of a computer model output.
data.f1 <- list(training = x1,fD = f1(x1), smooth = 1.99)

## Evaluation of GASP parameters
f1_MLEs = eval_GASP_RFP(data.f1,list(function(x){x^0},function(x){x^1}),1,FALSE)

## Evaluate the emulator
xn = seq(-1,1,.01)
GASP_type2_f1 <- eval_type2_GASP(as.matrix(xn),f1_MLEs)

par(mfrow = c(1,1))
par(mar = c(6.1, 6.1, 5.1, 2.1))
ylim = c(-1.5,1.5)
GASP_plot(GASP_type2_f1,f1,data.f1,"Type 2 GASP",ylab = " f",xlab = "x",
ylim = ylim, plot_training = TRUE)

s = GASP_type2_f1$mu
s.var = diag(GASP_type2_f1$var)

x2 = seq(-0.95,0.95,length = 6)#f1(x1)
data.f2 <- list(training = x2,fD = f2(x2), smooth = 2) # linking requires this emulator
## to have smoothness parameter equal to 2

f2_MLEs = eval_GASP_RFP(data.f2,list(function(x){x^0},function(x){x^1}),1,FALSE)

GASP_type1_f2 <- eval_type1_GASP(as.matrix(seq(-3.5,3.5,.01)),f2_MLEs)
GASP_type2_f2 <- eval_type2_GASP(as.matrix(seq(-1,1,.01)),f2_MLEs)
TGASP_f2 <- eval_TGASP(as.matrix(seq(-1,1,.01)),f2_MLEs)

ylim = c(-1.5,1.5)
# labels = c(expression(phantom(x)*phantom(x)*phantom(x)*f(x[1])),
# expression(f(x[2])*phantom(x)*phantom(x)*phantom(x)),
# expression(f(x[3])),expression(f(x[4])),
# expression(f(x[5])),expression(f(x[6])))

par(mar = c(6.1, 6.1, 5.1, 2.1))
GASP_plot(GASP_type2_f2,f2,data.f2, "Type 2 GASP",labels = x2,xlab= "z",ylab = " g",
ylim = ylim,plot_training = TRUE)

```

```

le <- link(f1_MLEs, f2_MLEs, as.matrix(xn))

## Construct an empirical emulator
n.samples = 100
em2.runs<-mat.or.vec(n.samples,length(s))
library(MASS)
for(i in 1:n.samples) {
  GASP = eval_type2_GASP(as.matrix(mvrnorm(1,s,diag(s.var))),f2_MLEs)
  em2.runs[i,] <- mvrnorm(1,GASP$mu, GASP$var)
}

## Plot the empirical GASP emulator
data.f2f1 <- list(training = x1,fD = f2f1(x1), smooth = 2)

par(mar = c(6.1, 6.1, 5.1, 2.1))
emp_GASP_plot(le$em2,f2f1,data.f2f1,"Linked",apply(em2.runs,2,quantile,probs = 0.025),
              apply(em2.runs,2,quantile,probs = 0.975),
              ylab = expression("g" ~ scriptscriptstyle(0) ~ "f"),xlab = "x, input",ylim = ylim)

```

eval_GASP_RFP

Evaluation of parameters of a Gaussian stochastic process emulator of a computer model.

Description

This function evaluates parameters of a Gaussian stochastic process emulator of a computer model based on a few observations which are available from the simulator of a computer model.

Usage

```
eval_GASP_RFP(data, basis, corr.cols, nugget)
```

Arguments

data	list which consists of three objects: training input values (which may be multi-variate, along several dimensions), corresponding output values of a simulator (scalar) and a vector of smoothness parameter(s) along each input direction.
basis	A set of functions in the mean of a Gaussian process. Typically assumed to be linear in one or several dimensions.
corr.cols	specifies which input directions must be included in the specification of a correlation function.
nugget	Parameter which accounts for possible small stochasticity in the output of a computer model. Default is FALSE.

Details

See examples which illustrate inputs specification to the function.

Value

Function returns a list of objects, including estimates of parameters, which is subsequently may be used for construction of a GASP approximation with the estimated parameters and the data involved.

delta	Estimates of range parameters in the correlation function.
eta	Estimates of a nugget.
sigma.sq	Estimates of variance.
data	Input parameter returned for convenience.
nugget	Input parameter returned for convenience.
basis	Input parameter returned for convenience.
corr.cols	Input parameter returned for convenience.

Author(s)

Ksenia N. Kzyurova, kseniak.ucoz.net.

References

Ksenia N. Kzyurova, James O. Berger, and Robert L. Wolpert. Coupling computer models through linking their statistical emulators. *SIAM/ASA Journal on Uncertainty Quantification*, 6(3): 1151-1171, 2018

Gu, M., Wang, X., Berger, J. O. et al. (2018) Robust Gaussian stochastic process emulation. *The Annals of Statistics*, 46, 3038-3066.

Examples

```
## Function f1 is a simulator
f1<-function(x){sin(pi*x)}

## One-dimensional inputs are x1
x1 <- seq(-1,1,.37)

## data.f1 contains the list of data inputs (training) and outputs (fD) together with the assumed
## fixed smoothness of a computer model output. This corresponds to the smoothness in a product
## power exponential correlation function used for construction of the emulator.
data.f1 <- list(training = x1,fD = f1(x1), smooth = 1.99)

## Evaluation of GASP parameters
f1_MLEs = eval_GASP_RFP(data.f1,list(function(x){x^0},function(x){x^1}),1,FALSE)
```

eval_TGASP

*T-GASP emulator***Description**

This function evaluates the third GASP of a computer model within objective Bayesian (OB) implementation of the GASP, resulting in T-GASP.

Usage

```
eval_TGASP(input, GASPparams)
```

Arguments

input	Input values (the same dimension as training input data in the next argument GASPparams)
GASPparams	The output of the function eval_GASP_RFP.

Value

Function returns a list of three objects

x	Inputs.
mu	Mean of an emulator.
var	Covariance matrix of an emulator.

Author(s)

Ksenia N. Kyzuyrova, kseniak.ucoz.net.

Examples

```
## Function f2 is a simulator
f2<-function(x){cos(5*x)}

## One-dimensional inputs x2
x2 = seq(-0.95,0.95,length = 6)
data.f2 <- list(training = x2,fD = f2(x2), smooth = 2)

## Evaluation of GASP parameters
f2_MLEs = eval_GASP_RFP(data.f2,list(function(x){x^0},function(x){x^1}),1,FALSE)

## Evaluation of a T-GASP emulator
TGASP_f2 <- eval_TGASP(as.matrix(seq(-1,1,.01)),f2_MLEs)
```

eval_type1_GASP	<i>The first type of an emulator of a computer model</i>
-----------------	--

Description

This function evaluates the first GASP of a computer model using maximum a posteriori estimates (MAP) of parameters of the GASP.

Usage

```
eval_type1_GASP(input, GASPparams)
```

Arguments

input	input values (the same dimension as training input data in the next argument GASPparams)
GASPparams	The output of the function eval_GASP_RFP.

Details

See examples which illustrate inputs specification to the function.

Value

Function returns a list of three objects

x	Inputs.
mu	Mean of an emulator.
var	Covariance matrix of an emulator.

Author(s)

Ksenia N. Kyzuyurova, kseniak.ucoz.net.

Examples

```
## Function f1 is a simulator
f1<-function(x){sin(pi*x)}

## One-dimensional inputs are x1
x1 <- seq(-1,1,.37)

## The following contains the list of data inputs (training) and outputs (fD) together with the
## assumed fixed smoothness of a computer model output.
data.f1 <- list(training = x1,fD = f1(x1), smooth = 1.99)

## Evaluation of GASP parameters
f1_MLEs = eval_GASP_RFP(data.f1,list(function(x){x^0},function(x){x^1}),1,FALSE)
```

```
## Evaluate the emulator
xn = seq(-1,1,.01)
GASP_type1_f1 <- eval_type1_GASP(as.matrix(xn),f1_MLEs)
```

eval_type2_GASP	<i>The second type of an emulator of a computer model</i>
-----------------	---

Description

This function evaluates the second GASP of a computer model within partial objective Bayesian (POB) implementation of the GASP.

Usage

```
eval_type2_GASP(input, GASPparams)
```

Arguments

input	input values (the same dimension as training input data in the next argument GASPparams)
GASPparams	The output of the function eval_GASP_RFP.

Details

See examples which illustrate inputs specification to the function.

Value

Function returns a list of three objects

x	Inputs.
mu	Mean of an emulator.
var	Covariance matrix of an emulator.

Author(s)

Ksenia N. Kyzuyrova, kseniak.ucoz.net.

Examples

```
## Function f2 is a simulator
f2<-function(x){cos(5*x)}

## One-dimensional inputs x2
x2 = seq(-0.95,0.95,length = 6)
data.f2 <- list(training = x2, fD = f2(x2), smooth = 2)
```



```
## Evaluation of GASP parameters
f2_MLEs = eval_GASP_RFP(data.f2,list(function(x){x^0},function(x){x^1}),1,FALSE)

## Evaluation of a second type GASP emulator
GASP_type2_f2 <- eval_type2_GASP(as.matrix(seq(-1,1,.01)),f2_MLEs)
```

GASP_plot

*Plot of the GASP***Description**

Function allows to plot the GASP in case of one-dimensional input.

Usage

```
GASP_plot(em, fun, data, emul_type, labels, yax, ylab, xlab,ylim,
col_CI_area,col_points,col_fun,col_mean,plot_training = FALSE, plot_fun = TRUE)
```

Arguments

em	the returned output from the function eval_type1_GASP(...) or eval_type2_GASP(...).
fun	Simulator function. Currently only one-dimensional input is supported.
data	Training data and smoothness. The same as supplied to eval_GASP_RFP(...) for construction of the GASP.
emul_type	A text string which provides description of an emulator.
labels	As in standard R plot.
yax	As in standard R plot.
ylab	As in standard R plot.
xlab	As in standard R plot.
ylim	As in standard R plot.
col_CI_area	Color of a credible area.
col_points	Color of the training points.
col_fun	Color of a simulator function.
col_mean	Color of the emulator of the GASP mean.
plot_training	(Not) to plot the training points. Default is FALSE.
plot_fun	(Not) to plot the simulator function. Default is TRUE.

Value

Plot

Note

The function requires further development to be automated for visualization along a single dimension out of multiple dimensions and along two dimensions out of multiple dimensions.

Author(s)

Ksenia N. Kyzyurova, kseniak.ucoz.net

Examples

```
## Function f1 is a simulator
f1<-function(x){sin(pi*x)}

## One-dimensional inputs are x1
x1 <- seq(-1,1,.37)

## The following contains the list of data inputs (training) and outputs (fD) together with the
## assumed fixed smoothness of a computer model output.
data.f1 <- list(training = x1,fD = f1(x1), smooth = 1.99)

## Evaluation of GASP parameters
f1_MLEs = eval_GASP_RFP(data.f1,list(function(x){x^0},function(x){x^1}),1,FALSE)

## Evaluate the emulator
xn = seq(-1,1,.01)
GASP_type1_f1 <- eval_type1_GASP(as.matrix(xn),f1_MLEs)

## Plot the emulator
par(mfrow = c(1,1))
par(mar = c(6.1, 6.1, 5.1, 2.1))
ylim = c(-1.5,1.5)
GASP_plot(GASP_type1_f1,fun = f1,data = data.f1,"",ylim = ylim, plot_training = TRUE)
```

[link](#)

Linking two emulators

Description

Function constructs a linked GASP emulator of a composite computer model $f_2(f_1)$.

Usage

```
link(f1_MLEs, f2_MLEs, test_input)
```

Arguments

f1_MLEs	Parameters of the emulator of a simulator f1.
f2_MLEs	Parameters of the emulator of a simulator f2.
test_input	Testing inputs.

Details

See examples which illustrate inputs specification to the function.

Value

Four types of the linked GASP.

em1	Type 1 emulator, which uses MAP estimates of parameters.
em2	Type 2 emulator within partial objective Bayesian (POB) implementation.
emT	T-GASP emulator within objective Bayesian (OB) implementation.
em3	Approximated T-GASP emulator with the Gaussian distribution.

Author(s)

Ksenia N. Kyzyurova, kseniak.ucoz.net

References

Ksenia N. Kyzyurova, James O. Berger, and Robert L. Wolpert. Coupling computer models through linking their statistical emulators. *SIAM/ASA Journal on Uncertainty Quantification*, 6(3): 1151-1171, 2018

Examples

```
## Function f1 is a simulator
f1<-function(x){sin(pi*x)}
## Function f2 is a simulator
f2<-function(x){cos(5*x)}

## Function f2(f1) is a simulator of a composite model
f2f1 <- function(x){f2(f1(x))}

## One-dimensional inputs are x1
x1 <- seq(-1,1,.37)

## The following contains the list of data inputs (training) and outputs (fD) together with
## the assumed fixed smoothness of a computer model output.
data.f1 <- list(training = x1,fD = f1(x1), smooth = 1.99)

## Evaluation of GASP parameters
f1_MLEs = eval_GASP_RFP(data.f1,list(function(x){x^0},function(x){x^1}),1,FALSE)

## Evaluate the emulator
xn = seq(-1,1,.01)
GASP_type2_f1 <- eval_type2_GASP(as.matrix(xn),f1_MLEs)

par(mfrow = c(1,1))
par(mar = c(6.1, 6.1, 5.1, 2.1))
ylim = c(-1.5,1.5)
GASP_plot(GASP_type2_f1,f1,data.f1,"Type 2 GASP",ylab = " f",xlab = "x",
```

```

ylim = ylim, plot_training = TRUE)

s = GASP_type2_f1$mu
s.var = diag(GASP_type2_f1$var)

x2 = seq(-0.95,0.95,length = 6)#f1(x1)
data.f2 <- list(training = x2,fD = f2(x2), smooth = 2) # linking requires this emulator
# to have smoothness parameter equal to 2

f2_MLEs = eval_GASP_RFP(data.f2,list(function(x){x^0},function(x){x^1}),1,FALSE)

GASP_type1_f2 <- eval_type1_GASP(as.matrix(seq(-3.5,3.5,.01)),f2_MLEs)
GASP_type2_f2 <- eval_type2_GASP(as.matrix(seq(-1,1,.01)),f2_MLEs)
TGASP_f2 <- eval_TGASP(as.matrix(seq(-1,1,.01)),f2_MLEs)

ylim = c(-1.5,1.5)
# labels = c(expression(phantom(x)*phantom(x)*phantom(x)*f(x[1])),
# expression(f(x[2])*phantom(x)*phantom(x)*phantom(x)),
# expression(f(x[3])),expression(f(x[4])),
# expression(f(x[5])),expression(f(x[6])))

par(mar = c(6.1, 6.1, 5.1, 2.1))
GASP_plot(GASP_type2_f2,f2,data.f2, "Type 2 GASP",labels = x2,xlab= "z",ylab = " g",
ylim = ylim,plot_training = TRUE)

le <- link(f1_MLEs, f2_MLEs, as.matrix(xn))

## Plot second type of the linked GASP
data.f2f1 <- list(training = x1,fD = f2f1(x1), smooth = 2)

par(mar = c(6.1, 6.1, 5.1, 2.1))
GASP_plot(le$em2,f2f1,data.f2f1,"Linked",labels = x1,
ylab = expression("g" ~ scriptscriptstyle(0) ~ "f"),xlab = "x",ylim = ylim)

```

NGASPmetrics

GASP performance assessment measures

Description

Evaluates frequentist performance of the GASP.

Usage

```
NGASPmetrics(GASP, true_output, ref_output)
```

Arguments

GASP	GASP emulator.
true_output	Output from the simulator.
ref_output	Heuristic emulator output.

Value

List of performance measures.

RMSPE_base	Root mean square predictive error with respect to the heuristic emulator output.
RMSPE	Root mean square predictive error for the emulator output
ratio	ratio of RMSPE_base to RMSPE. Ratio = RMSPE_base/RMSPE
CIs	95% central credible intervals
emp_cov	95% empirical coverage within the CIs
length_CIs	Average length of 95% central credible intervals

Author(s)

Ksenia N. Kzyurova, ksenia.ucoz.net

References

Ksenia N. Kzyurova, James O. Berger, and Robert L. Wolpert. Coupling computer models through linking their statistical emulators. *SIAM/ASA Journal on Uncertainty Quantification*, 6(3): 1151-1171, 2018

Examples

```
## Function f1 is a simulator
f1<-function(x){sin(pi*x)}

## One-dimensional inputs are x1
x1 <- seq(-1,1,.37)

## The following contains the list of data inputs (training) and outputs (fD) together with
## the assumed fixed smoothness of a computer model output.
data.f1 <- list(training = x1,fD = f1(x1), smooth = 1.99)

## Evaluation of GASP parameters
f1_MLEs = eval_GASP_RFP(data.f1,list(function(x){x^0},function(x){x^1}),1,FALSE)

## Evaluate the emulator
xn = seq(-1,1,.01)
GASP_type2_f1 <- eval_type2_GASP(as.matrix(xn),f1_MLEs)

## Plot the emulator
par(mar = c(6.1, 6.1, 5.1, 2.1))
GASP_plot(GASP_type2_f1,data = data.f1,emul_type = "",ylim = ylim, plot_training = TRUE)

## Measure performance of an emulator
NGASPMetrics(GASP_type2_f1,f1(xn),mean(f1(xn)))
```

TGASPmetrics

Performance measurement of a T-GASP

Description

Evaluates frequentist performance of a T-GASP.

Usage

```
TGASPmetrics(TGASP, true_output, ref_output)
```

Arguments

TGASP	TGASP emulator (in the paper this is done within an objective Bayesian implementation - OB emulator.)
true_output	Output from the simulator.
ref_output	Heuristic emulator output.

Details

See examples which illustrate the use of the function.

Value

List of performance measures.

RMSPE_base	Root mean square predictive error with respect to the heuristic emulator output.
RMSPE	Root mean square predictive error for the emulator output
ratio	ratio of RMSPE_base to RMSPE. Ratio = RMSPE_base/RMSPE
CIs	95% central credible intervals
emp_cov	95% empirical coverage within the CIs
length_CIs	Average length of 95% central credible intervals

Author(s)

Ksenia N. Kzyurova, ksenia.ucoz.net

References

Ksenia N. Kzyurova, James O. Berger, and Robert L. Wolpert. Coupling computer models through linking their statistical emulators. *SIAM/ASA Journal on Uncertainty Quantification*, 6(3): 1151-1171, 2018

Examples

```
## Function f1 is a simulator
f1<-function(x){sin(pi*x)}

## One-dimensional inputs are x1
x1 <- seq(-1,1,.37)

## The following contains the list of data inputs (training) and outputs (fD) together with
## the assumed fixed smoothness of a computer model output.
data.f1 <- list(training = x1,fD = f1(x1), smooth = 1.99)

## Evaluation of GASP parameters
f1_MLEs = eval_GASP_RFP(data.f1,list(function(x){x^0},function(x){x^1}),1,FALSE)

## Evaluate the emulator
xn = seq(-1,1,.01)
TGASP_f1 <- eval_TGASP(as.matrix(xn),f1_MLEs)

## Plot the emulator
par(mfrow = c(1,1))
par(mar = c(6.1, 6.1, 5.1, 2.1))
ylim = c(-1.5,1.5)
TGASP_plot(TGASP_f1,f1,data.f1,ylim = ylim)

## Measure the performance of the emulator
TGASPmetrics(TGASP_f1,f1(xn),mean(f1(xn)))
```

TGASP_plot

*T-GASP plot***Description**

Function allows to plot the TGASP in case of one-dimensional input. Black-and-white version.

Usage

```
TGASP_plot(tem, fun, data, labels, ylim, points)
```

Arguments

tem	TGasP emulator.
fun	Simulator function.
data	Training data and smoothness. The same as supplied to eval_GASP_RFP(...) for construction of a GASP.
labels	As in standard R plot.
ylim	As in standard R plot.
points	(Not) to plot the training points.

Details

See examples.

Value

Plot

Note

The function requires further development to be automated for visualization along a single dimension out of multiple dimensions and along two dimensions out of multiple dimensions.

This function needs to be automated to allow for fast visualization of a single emulator (with no comparison to the actual simulator function), etc.

Author(s)

Ksenia N. Kyzuyurova, kseniak.ucoz.net

Examples

```
## Function f1 is a simulator
f1<-function(x){sin(pi*x)}

## One-dimensional inputs are x1
x1 <- seq(-1,1,.37)

## The following contains the list of data inputs (training) and outputs (fD) together with
## the assumed fixed smoothness of a computer model output.
data.f1 <- list(training = x1,fD = f1(x1), smooth = 1.99)

## Evaluation of GASP parameters
f1_MLEs = eval_GASP_RFP(data.f1,list(function(x){x^0},function(x){x^1}),1,FALSE)

## Evaluate the emulator
xn = seq(-1,1,.01)
TGASP_f1 <- eval_TGASP(as.matrix(xn),f1_MLEs)

## Plot the emulator
par(mfrow = c(1,1))
par(mar = c(6.1, 6.1, 5.1, 2.1))
ylim = c(-1.5,1.5)
TGASP_plot(TGASP_f1,f1,data.f1,ylim = ylim)
```


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