Package 'FLASHMM'

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Title Fast and Scalable Single Cell Differential Expression Analysis
using Mixed-Effects Models

Version 1.2.1

Description A fast and scalable linear mixed-effects model (LMM) estimation algorithm
for analysis of single-cell differential expression. The algorithm uses
summary-level statistics and requires less computer memory to fit the LMM.

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Imports stats, MASS, Matrix

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Contents

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contrast.matrix   *Construct Contrast Matrix*

Description

Construct the contrast matrix to make various comparsions of different treatments.

Usage

```
contrast.matrix(contrast, model.matrix.names)
```

Arguments

contrast   A vector of character strings specifying the various comparisons, which are the
                  expressions constituted by model.matrix.names.

model.matrix.names
                  Column names of model (design) matrix.

Value

Matrix which columns correspond to contrasts.

Examples

```
model_variables <- c("A", "B", "C", "D")
contrast <- c("AvsB" = "A-B", "AvsC" = "A-C", 'AvsB.C.D'= "A-(B+C+D)/3")
contrast.matrix(contrast, model_variables)
```

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lmm   *Fitting Linear Mixed-effects Models*

Description

lmm is used to fit linear mixed-effects models (LMM) based on summary-level data. The LMM parameters are estimated by restricted maximum likelihood (REML) with Fisher scoring (FS) gradient descent algorithm.

Usage

```
lmm(
  XX,
  XY,
  ZX,
  ZY,
  ZZ,
```

```
    Ynorm,
    n,
    d = ncol(ZZ),
    theta0 = NULL,
    method = c("REML", "ML"),
    max.iter = 50,
    epsilon = 1e-05,
    output.cov = TRUE,
    output.RE = FALSE
)
```

Arguments

XX	t(X)%*%X, X is a design matrix for fixed effects.
XY	t(Y%*%X), Y is a features-by-samples matrix of observed responses (genes-by-cells expression matrix for scRNA-seq).
ZX	t(Z)%*%X, Z = [Z1, ..., Zk], a design matrix for k random factors (variables).
ZY	t(Y%*%Z).
ZZ	t(Z)%*%Z.
Ynorm	Norms for features (each row in Y), that is, Ynorm = rowSums(Y*Y).
n	Numbers of samples (cells in scRNA-seq), nrow(X).
d	A vector of (m1,...,mk), mi = ncol(Zi), number of columns in Zi. m1 + ... + mk = ncol(Z), number of columns in Z.
theta0	A vector of initial values of the variance components, (s1, ...,sk, s_(k+1)), si = sigma_i^2, the variance component of the i-th type random effects. s_(k+1) = sigma^2, the variance component of model residual error.
method	Either REML or ML with Fisher scoring (FS) iterative algorithm.
max.iter	The maximal number of iterations for the iterative algorithm.
epsilon	Positive convergence tolerance. If the absolute value of the first partial derivative of log likelihood is less than epsilon, the iterations converge.
output.cov	If TRUE, output the covariance matrices for the estimated coefficients, which are needed for testing contrasts.
output.RE	If TRUE, output the best linear unbiased prediction (BLUP) of the random effects.

Value

A list containing the following components:

dlogL First partial derivatives of log-likelihoods for each feature (gene).

logLik Either log maximum likelihood (ML) or log restricted maximum likelihood (REML) for each feature (gene). The log restricted maximum likelihood may have a constant difference from other computations. The constant difference is related to the fixed design matrix, X.

niter Nmbers of iterations for each feature (gene).

coef A matrix of estimated coefficients (fixed effects), each column corresponds to a feature (gene) and each row one covariate.

se A matrix of the standard errors of the estimated coefficients.

t A matrix of t-values for the fixed effects, equal to coef/se.

df Degrees of freedom.

p A matrix of two-sided p-values for the fixed effects.

cov A array of covariance matrices of the estimated coefficients (fixed effects).

theta A matrix of the estimated variance components, each column corresponds to a feature (gene) and each row one variance component. The last row is the variance component of the residual error.

se.theta Standard errors of the estimated theta.

RE A matrix of the best linear unbiased prediction (BLUP) of random effects.

Examples

```
#Generate data: X, Y, and Z.
set.seed(2024)

n <- 1e3
m <- 10
Y <- matrix(rnorm(m*n), m, n)
rownames(Y) <- paste0("Gene", 1:nrow(Y))

trt <- sample(c("A", "B"), n, replace = TRUE)
X <- model.matrix(~ 0 + trt)

q <- 20
sam <- rep(NA, n)
sam[trt == "A"] <- paste0("A", sample.int(q/2, sum(trt == "A"), replace = TRUE))
sam[trt == "B"] <- paste0("B", sample.int(q/2, sum(trt == "B"), replace = TRUE))
Z <- model.matrix(~ 0 + sam)
d <- ncol(Z)

#Fit LMM by summary-level data
#Compute and store the summary-level data:
n <- nrow(X)
XX <- t(X)%*%X
XY <- t(Y%*%X)
ZX <- t(Z)%*%X
ZY <- t(Y%*%Z)
ZZ <- t(Z)%*%Z
Ynorm <- rowSums(Y*Y)
fit <- lmm(XX, XY, ZX, ZY, ZZ, Ynorm = Ynorm, n = n, d = d)
str(fit)
```

---

lmmfit *Fitting Linear Mixed-effects Models*

Description

lmmfit, a wrapper function of lmm, fits linear mixed-effects models (LMM) by sample-level data. The LMM parameters are estimated by restricted maximum likelihood (REML) with Fisher scoring (FS) gradient descent algorithm.

Usage

```
lmmfit(
  Y,
  X,
  Z,
  d,
  theta0 = NULL,
  nBlocks = ceiling(nrow(Y) * ncol(Y) * 1e-08),
  method = c("REML", "ML"),
  max.iter = 50,
  epsilon = 1e-05,
  output.cov = TRUE,
  output.RE = FALSE
)
```

Arguments

Y	A features-by-samples matrix of responses (genes-by-cells matrix of gene expressions for scRNA-seq).
X	A design matrix for fixed effects, with rows corresponding to the columns of Y.
Z	A design matrix for random effects, with rows corresponding to the columns of Y. Z = [Z1, ..., Zk], and Zi, i=1,...,k, is the design matrix for the i-th type random factor.
d	A vector of (m1,...,mk), mi = ncol(Zi), number of columns in Zi. m1 + ... + mk = ncol(Z), number of columns in Z.
theta0	A vector of initial values of the variance components, (s1, ...,sk, s_(k+1)), si = sigma_i^2, the variance component of the i-th type random effects. s_(k+1) = sigma^2, the variance component of model residual error.
nBlocks	Number of blocks, used for blocking a big data to reduce the storage required in computing.
method	The REML with Fisher scoring (FS) iterative algorithm, REML-FS.
max.iter	The maximal number of iterations for the iterative algorithm.
epsilon	Positive convergence tolerance. If the absolute value of the first partial derivative of log likelihood is less than epsilon, the iterations converge.

| output.cov | If TRUE, output the covariance matrices for the estimated coefficients, which are needed for testing contrasts. |
| output.RE | If TRUE, output the best linear unbiased prediction (BLUP) of the random effects. |

Value

A list containing the following components:

dlogL First partial derivatives of log-likelihoods for each feature (gene).

logLik Maximum log-likelihoods (ML) or log-restricted-likelihood (REML) for each feature (gene).

niter Nmbers of iterations for each feature (gene).

coef A matrix of estimated coefficients (fixed effects), each column corresponds to a feature (gene) and each row one covariate.

se A matrix of the standard errors of the estimated coefficients.

t A matrix of t-values for the fixed effects, equal to coef/se.

df Degrees of freedom.

p A matrix of two-sided p-values for the fixed effects.

cov A array of covariance matrices of the estimated coefficients (fixed effects).

theta A matrix of the estimated variance components, each column corresponds to a feature (gene) and each row one variance component. The last row is the variance component of the residual error.

se.theta Standard errors of the estimated theta.

RE A matrix of the best linear unbiased prediction (BLUP) of random effects.

Examples

```
#Generate data: X, Y, and Z.
set.seed(2024)

n <- 1e3
m <- 10
Y <- matrix(rnorm(m*n), m, n)
rownames(Y) <- paste0("Gene", 1:nrow(Y))

trt <- sample(c("A", "B"), n, replace = TRUE)
X <- model.matrix(~ 0 + trt)

q <- 20
sam <- rep(NA, n)
sam[trt == "A"] <- paste0("A", sample.int(q/2, sum(trt == "A"), replace = TRUE))
sam[trt == "B"] <- paste0("B", sample.int(q/2, sum(trt == "B"), replace = TRUE))
Z <- model.matrix(~ 0 + sam)
d <- ncol(Z)

#Fit LMM by the cell-level data
fit <- lmmfit(Y, X, Z, d = d)
str(fit)
```

```
#Hypothesis testing
lmmtest(fit)
lmmtest(fit, index = 2)
lmmtest(fit, contrast = cbind("B-A" = c(-1, 1)))
```

---

lmmtest   *Testing Fixed Effects and Contrasts of the Fixed Effects*

Description

lmmtest is used to test fixed effects or contrasts of fixed effects.

Usage

```
lmmtest(
  fit,
  index,
  contrast = NULL,
  alternative = c("two.sided", "less", "greater")
)
```

Arguments

fit	A result of lmmfit/lmm, which contains coef (estimates of fixed effects), a matrix with rows representing the fixed effects and columns the different response variables in the model, cov (covariance matrix of the fixed effects), an array of three dimensions for different response variables in the model, df (residual degree of freedom in the linear model).
index	A vector of integers or characters indicating which fixed effects are to be tested. By default index consists of all of the fixed effects. Ignored if contrast is not NULL.
contrast	A matrix with columns corresponding to contrasts of the fixed effects to be tested.
alternative	A character string specifying the alternative hypothesis, one of "two.sided", "greater" or "less".

Value

A matrix of coefficients, t-values and p-values, in which the rows correspond to the features (genes) and the columns the fixed effects (covariates). .

| simuRNAseq | *Simulating Multi-sample Multi-cell-type scRNA-seq Dataset based on Negative Binomial Distribution* |

Description

simuRNAseq simulates scRNA-seq data with multiple subjects (samples), multiple clusters (cell-types) and two treatments (conditions) based on a negative binomial (NB) distribution using a reference data as background or control. The reference data consisting of genes-by-cells counts matrix is used to estimate the NB dispersion and means for the genes to be simulated. The simulated genes are randomly selected from the reference data. The NB dispersion are estimated by the method-of-moments estimate (MME). The NB means for the background in the control are estimated by the sample mean. The NB means for the differentially expressed (DE) genes are given by the sample mean plus a log-fold change (logFC). The simulated cells are randomly selected from the meta data that specifies subjects, cell-types and treatments for the cells. The meta data consists of samples, clusters of cell types, and treatments, which can be generated either from reference data or randomly. If not provided, it will be randomly generated. A random seed is recommended to be specified by set.seed before simulating data.

Usage

```
simuRNAseq(
  counts,
  nGenes = nrow(counts),
  nCells = ncol(counts),
  metadata = NULL,
  samples.nested = TRUE,
  nsam = 25,
  ncls = 10,
  ntrt = 2,
  trt = NULL,
  nDEgenes = ncls,
  nDEgenesType,
  pDEgenesType = NULL,
  adjust.library.size = TRUE,
  direction = c("both", "up", "down"),
  minbeta = 0.25,
  maxbeta = 1,
  var.randomeffects = 0.1
)
```

Arguments

counts	A genes-by-cells matrix of reference counts. If missing, counts is generated by a negative binomial distribution.
nGenes	Number of genes to be simulated.

nCells	Number of cells to be simulated.
metadata	The meta data consisting of 4 columns: sam (sample labels), cls (cluster lables of cell types), trt (treatments or conditions) and libsize (library size or total counts per cell), which is randomly generated if not provided.
samples.nested	If TRUE, when metadata is not provided, each simulated subject (sample) belongs to only one condition (either treatment or control), that is, the subject is nested in a condition (treatment).
nsam	Number of subjects (individuals).
ncls	Number of clusters (cell-types).
ntrt	Number of treatments (only one condition is treated).
trt	Treatment, specifying which condition is treated.
nDEgenes	Total number of DE genes.
nDEgenesType	Number of DE genes specific to a cell type, named by cell cluster labels.
pDEgenesType	Proportion of DE genes in a cell-type. Default NULL means equal proportion.
adjust.library.size	
	If TRUE, adjust library sizes using the reference counts.
direction	Specify if the DE genes are up- and/or down-regulated.
minbeta	Lower bound of the DE gene logFC.
maxbeta	Upper bound of the DE gene logFC. minbeta < maxbeta. If direction = "both", minbeta*maxbeta > 0. If direction = "up", minbeta > 0. If direction = "down", maxbeta < 0.
var.randomeffects	
	Variance of random effects

Value

ref.mean.dispersion: A data frame of the reference counts' means and dispersion,

metadata: Meta data consisting of 4 columns: sam (sample labels), cls (cluster lables of cell types), trt (two treatments/conditions) and libsize (library sizes).

counts: A genes-by-cells matrix of the simulated counts.

DEgenes: A data frame of DE genes consisting of 3 columns: gene, beta (effect), and cluster to which the gene is specific.

treatment: The condition treated.

Examples

```
#Simulate a multi-sample multi-cell-type scRNA-seq dataset.
set.seed(2412)
dat <- simuRNAseq(nGenes = 50, nCells = 1000, nsam = 25, ncls = 4, ntrt = 2, nDEgenes = 6)
str(dat)
table(dat$metadata[, c("sam", "trt")]) #The samples are nested in a condition.

#Analyze differentially expressed (DE) genes specific to a cell-type using LMM.
Y <- log(dat$counts + 1) #expressions (log-transformed counts)
```

```
X <- model.matrix(~ 0 + log(libsize) + cls + cls:trt, data = dat$metadata)
Z <- model.matrix(~ 0 + sam, data = dat$metadata)
d <- ncol(Z)

#Fit LMM using cell-level data.
fit <- lmmfit(Y, X, Z, d = d)

#Fit LMM using summary-level data.
#Compute and store the summary-level data:
n <- nrow(X)
XX <- t(X)%*%X
XY <- t(Y%*%X)
ZX <- t(Z)%*%X
ZY <- t(Y%*%Z)
ZZ <- t(Z)%*%Z
Ynorm <- rowSums(Y*Y)
fitss <- lmm(XX, XY, ZX, ZY, ZZ, Ynorm = Ynorm, n = n, d = d)
identical(fit, fitss)

#Hypothesis testing
test <- lmmtest(fit)
head(test)

#The DE genes specific to a cell-type.
tail(test[, grep(":", colnames(test))])
#The true DE genes
dat$DEgenes
```

---

sslmm   *Computing Summary-level Data from Individual-level Data*

Description

sslmm can be used to compute the correlation-related summary statistics (summary-level data) for lmm function.

Usage

```
sslmm(X, Y, Z)
```

Arguments

X	A design matrix for fixed effects, with rows corresponding to the columns of Y.
Y	A features-by-samples matrix of responses (genes-by-cells matrix of gene expressions for scRNA-seq).
Z	A design matrix for random effects, with rows corresponding to the columns of Y.

Value

A list of summary statistics: XX = t(X)%*%X, XY = t(X)%*%t(Y), ZX = t(Z)%*%X, ZY = t(Z)%*%t(Y), ZZ = t(Z)%*%Z, Ynorm = rowSums(Y*Y) and n = nrow(X).

Examples

```
n <- 1e3
set.seed(2024)
p <- 2
X <- matrix(rnorm(p*n), n, p)
colnames(X) <- paste0("X", 1:ncol(X))
m <- 3
Y <- matrix(rnorm(m*n), m, n)
rownames(Y) <- paste0("Y", 1:nrow(Y))
q <- 4
Z <- gl(q, n/q, labels = letters[1:q])
Z <- model.matrix(~ 0 + Z)
sslmm(X, Y, Z)
```

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