Package 'ClaimsProblems'

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Type Package

Title Analysis of Conflicting Claims

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Description The analysis of conflicting claims arises when an amount has to be divided among a set of agents with claims that exceed what is available. A rule is a way of selecting a division among the claimants. This package computes the main rules introduced in the literature from ancient times to the present. The inventory of rules covers the proportional and the adjusted proportional rules, the constrained equal awards and the constrained equal losses rules, the constrained egalitarian, the Piniles' and the minimal overlap rules, the random arrival and the Talmud rules. Besides, the Dominguez and Thomson and the average-of-awards rules are also included. All of them can be found in the book by W. Thomson (2019), How to divide when there isn't enough. From Aristotle, the Talmud, and Maimonides to the axiomatics of resource allocation', except for the average-of-awards rule, introduced by Mirás Calvo et al. (2022), <doi:10.1007/s00355-022-01414-6>. In addition, graphical diagrams allow the user to represent, among others, the set of awards, the paths of awards, the schedules of awards of a rule, and some indexes. A good understanding of the similarities and differences between the rules is useful for better decision-making. Therefore, this package could be helpful to students, researchers, and managers alike. For a more detailed explanation of the package, see Mirás Calvo et al. (2023), <doi:10.1016/j.dajour.2022.100160>.

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Contents

`AA` *Average-of-awards rule*

Description

This function returns the awards vector assigned by the average-of-awards rule (AA) to a claims problem.

Usage

```
AA(E, d, name = FALSE)
```

Arguments

`E`	The endowment.
`d`	The vector of claims.
`name`	A logical value.

Details

Let $N = \{1, \ldots, n\}$ be the set of claimants, $E \geq 0$ the endowment to be divided and $d \in \mathbb{R}_+^N$ the vector of claims such that $\sum_{i \in N} d_i \geq E$.

A vector $x = (x_1, \ldots, x_n)$ is an awards vector for the claims problem (E, d) if $0 \leq x \leq d$ and satisfies the balance requirement, that is, $\sum_{i=1}^n x_i = E$. Let $X(E, d)$ be the set of awards vectors for (E, d).

The average-of-awards rule (AA) assigns to each claims problem (E, d) the expectation of the uniform distribution defined over the set of awards vectors, that is, the centroid of $X(E, d)$.

Let μ be the $(n-1)$-dimensional Lebesgue measure and $V(E, d) = \mu(X(E, d))$ be the measure (volume) of the set of awards $X(E, d)$. The average-of-awards rule assigns to each claims problem (E, d) the awards vector given by:

$$\text{AA}(E, d) = \frac{1}{V(E, d)} \int_{X(E,d)} x d\mu.$$

The average-of-awards rule corresponds to the core-center solution of the associated coalitional (pessimistic) game.

The function AA is programmed with the algorithm of Mirás Calvo et al. (2024b), which is an improved version of the algorithm of Mirás Calvo et al. (2024a).

Value

The awards vector selected by the AA rule. If `name = TRUE`, the name of the function (AA) as a character string.

References

Gonzalez-Díaz, J. and Sánchez-Rodríguez, E. (2007). A natural selection from the core of a TU game: the core-center. International Journal of Game Theory 36(1), 27-46.

Mirás Calvo, M.A., Núñez Lugilde, I., Quinteiro Sandomingo, C., and Sánchez-Rodríguez, E. (2024a). An algorithm to compute the average-of-awards rule for claims problems with an application to the allocation of CO_2 emissions. Annals of Operations Research 336, 1435-1459.

Mirás Calvo, M.A., Núñez Lugilde, I., Quinteiro Sandomingo, C., and Sánchez-Rodríguez, E. (2024b). On properties of the set of awards vectors for a claims problem. TOP 32, 137-167.

Mirás Calvo, M.A., Quinteiro Sandomingo, C., and Sánchez-Rodríguez, E. (2022). The average-of-awards rule for claims problems. Social Choice and Welfare 59, 863-888.

See Also

allrules, axioms, CD, coalitionalgame, setofawards, volume.

Examples

```
E=10
d=c(2,4,7,8)
AA(E,d)
#The average-of-awards rule is self-dual: AA(E,d)=d-AA(D-E,d)
D=sum(d)
d-AA(D-E,d)
```

`allrules` *Summary of the division rules*

Description

This function returns the awards vectors selected, for a given claims problem, by the rules: AA, APRO, CE, CEA, CEL, AV, DT, MO, PIN, PRO, RA, Talmud, and RTalmud.

Usage

```
allrules(E, d, draw = TRUE, col = NULL)
```

Arguments

`E`	The endowment.
`d`	The vector of claims.
`draw`	A logical value.
`col`	The colours (useful only if `draw=TRUE`). If `col=NULL` then the sequence of default colours is: c("red", "blue", "green", "yellow", "pink", "orange", "coral4", "darkgray", "burlywood3", "black", "darkorange", "darkviolet","darkgreen").

Details

Let $N = \{1, \ldots, n\}$ be the set of claimants, $E \geq 0$ the endowment to be divided and $d \in \mathbb{R}_+^N$ the vector of claims such that $\sum_{i \in N} d_i \geq E$.

A vector $x = (x_1, \ldots, x_n)$ is an awards vector for the claims problem (E, d) if $0 \leq x \leq d$ and satisfies the balance requirement, that is, $\sum_{i=1}^{n} x_i = E$.

A rule is a function that assigns to each claims problem (E, d) an awards vector. The formal definitions of the main rules are given in the corresponding function help.

Value

A data-frame with the awards vectors selected by the main division rules. If `draw = TRUE`, it displays a mosaic plot representing the data-frame.

References

Mirás Calvo, M.A., Quinteiro Sandomingo, C., and Sánchez-Rodríguez, E. (2022). The average-of-awards rule for claims problems. Social Choice and Welfare 59, 863-888.

Thomson, W. (2019). How to divide when there isn't enough. From Aristotle, the Talmud, and Maimonides to the axiomatics of resource allocation. Cambridge University Press.

See Also

AA, APRO, axioms, CD, CE, CEA, CEL, AV, DT, MO, PIN, PRO, RA, Talmud, RTalmud, verticalruleplot.

Examples

```
E=10
d=c(2,4,7,8)
allrules(E,d)
```

APRO *Adjusted proportional rule*

Description

This function returns the awards vector assigned by the adjusted proportional rule (APRO) to a claims problem.

Usage

```
APRO(E, d, name = FALSE)
```

Arguments

E	The endowment.
d	The vector of claims.
name	A logical value.

Details

Let $N = \{1, \ldots, n\}$ be the set of claimants, $E \geq 0$ the endowment to be divided and $d \in \mathbb{R}_+^N$ the vector of claims such that $\sum_{i \in N} d_i \geq E$. For each coalition $S \in 2^N$, let $d(S) = \sum_{j \in S} d_j$ and $N \backslash S$ be the complementary coalition of S.

The minimal right of claimant $i \in N$ in (E, d) is whatever is left after every other claimant has received his claim, or 0 if that is not possible:

$$m_i(E, d) = \max\{0, E - d(N \backslash \{i\})\},\ i = 1, \ldots, n.$$

Let $m(E, d) = (m_1(E, d), \ldots, m_n(E, d))$ be the vector of minimal rights.

The adjusted proportional rule (APRO) first assigns to each claimant its minimal right, and then divides the remainder of the endowment $E' = E - \sum_{i=1}^n m_i(E, d)$ proportionally with respect to the new claims. The vector of the new claims d' is determined by the minimum of the remainder and the lowered claims, $d'_i = \min\{E - \sum_{j=1}^n m_j(E, d), d_i - m_i\},\ i = 1, \ldots, n$. Therefore,

$$\text{APRO}(E, d) = m(E, d) + \text{PRO}(E', d').$$

The adjusted proportional rule corresponds to the τ-value of the associated (pessimistic) coalitional game.

Value

The awards vector selected by the APRO rule. If `name = TRUE`, the name of the function (APRO) as a character string.

References

Curiel, I. J., Maschler, M., and Tijs, S. H. (1987). Bankruptcy games. Zeitschrift für operations research 31(5), A143-A159.

Mirás Calvo, M.Á., Núñez Lugilde, I., Quinteiro Sandomingo, C., and Sánchez-Rodríguez, E. (2023). Refining the Lorenz-ranking of rules for claims problems on restricted domains. International Journal of Economic Theory 19(3), 526-558.

Thomson, W. (2019). How to divide when there isn't enough. From Aristotle, the Talmud, and Maimonides to the axiomatics of resource allocation. Cambridge University Press.

See Also

allrules, axioms, CD, coalitionalgame, PRO.

Examples

```
E=10
d=c(2,4,7,8)
APRO(E,d)
#The adjusted proportional rule is self-dual: APRO(E,d)=d-APRO(D-E,d)
D=sum(d)
d-APRO(D-E,d)
```

AV *Average rule*

Description

This function returns the awards vector assigned by the average rule (AV) to a claims problem.

Usage

```
AV(E, d, name = FALSE)
```

Arguments

E	The endowment.
d	The vector of claims.
name	A logical value.

Details

Let $N = \{1, \ldots, n\}$ be the set of claimants, $E \geq 0$ the endowment to be divided and $d \in \mathbb{R}_+^N$ the vector of claims such that $\sum_{i \in N} d_i \geq E$.

The average rule (AV) is the average of constrained equal awards (CEA) and constrained equal losses (CEL) rules. That is,

$$\text{AV}(E, d) = \frac{\text{CEA}(E, d) + \text{CEL}(E, d)}{2}.$$

Value

The awards vector selected by the AV rule. If `name = TRUE`, the name of the function (AV) as a character string.

References

Thomson, W. (2019). How to divide when there isn't enough. From Aristotle, the Talmud, and Maimonides to the axiomatics of resource allocation. Cambridge University Press.

See Also

allrules, axioms, CEA, CEL, Talmud, RTalmud.

Examples

```
E=10
d=c(2,4,7,8)
AV(E,d)
```

`axioms` *Properties of the rules*

Description

This function shows which of the main properties the rules satisfy.

Usage

```
axioms(Rules = "All", Properties = "All")
```

Arguments

`Rules`	The rules: AA, APRO, CE, CEA, CEL, AV, MO, PIN, PRO, RA, Talmud. By default, `Rules = "All"`.
`Properties`	The properties listed in the description section. By default, `Properties = "All"`.

Details

Let $\mathcal{N}$ be the set of all finite nonempty subsets of the natural numbers $\mathbb{N}$. Let $N = \{1, \ldots, n\}$ be the set of claimants, $E \geq 0$ the endowment to be divided and $d \in \mathbb{R}_+^N$ the vector of claims such that $D = \sum_{i \in N} d_i \geq E$. We denote the class of claims problems with set of claimants N by C^N. Given $z \in \mathbb{R}^N$, $S \in 2^N$ and $N' \subset N \in \mathcal{N}$, let $z(S) = \sum_{j \in S} z_j$, $z_{N'} = (z_i)_{i \in N'}$ and denote $z_{-i} = z_{N \setminus \{i\}} \in \mathbb{R}^{N \setminus \{i\}}$. For simplicity, we will write $z = (z_{-i}, z)$.

The minimal right of claimant $i \in N$ in (E, d) is defined by $m_i(E, d) = \max\{0, E - d(N \setminus \{i\})\}$. Let $m(E, d) = (m_1(E, d), \ldots, m_n(E, d))$ be the vector of minimal rights.

The truncated claim of claimant $i \in N$ in (E, d) is defined by $t_i(E, d) = \min\{d_i, E\}$. Let $t(E, d) = (t_1(E, d), \ldots, t_n(E, d))$ be the vector of truncated claims.

Given $(E, d) \in C^N$ and $k \in \mathbb{N}$, we say that (E', d') is a k-replica of (E, d) if $E' = kE$, $N' \supset N$, $|N'| = k|N|$, and there is a partition $(N^i)_{i \in N}$ of N' such that for each $i \in N$ and each $j \in N^i$, $|N^i| = k$ and $d'_j = d_i$.

A vector $x = (x_1, \ldots, x_n)$ is an awards vector for the claims problem $(E, d) \in C^N$ if $0 \leq x \leq d$ and satisfies the balance requirement, that is, $x(N) = E$. Let $X(E, d)$ be the set of awards vectors for the problem (E, d). A rule is a function that assigns to each claims problem (E, d) an awards vector.

A rule $\mathcal{R}$ satisfies:

1) **Anonymity**, if for each $(E, d) \in C^N$, each bijection f from N into itself, and each $i \in N$, we have that $\mathcal{R}_i(E, d) = \mathcal{R}_{f(i)}(E, (d_{f(i)})_{i \in N})$.

2) **Continuity**, if for each sequence $\{(E^\ell, d^\ell)\}$ of elements of C^N and each $(E, d) \in C^N$, if $(E^\ell, d^\ell) \to (E, d)$ then $\mathcal{R}(E^\ell, d^\ell) \to \mathcal{R}(E, d)$.

3) **Homogeneity**, if for each $(E, d) \in C^N$ and each $\rho > 0$, then $\mathcal{R}(\rho E, \rho d) = \rho \mathcal{R}(E, d)$.

4) $\mathbf{\frac{1}{|N|}}$**-truncated-claims lower bounds on awards**, if for each $(E, d) \in C^N$, then $\mathcal{R}(E, d) \geq \frac{1}{|N|} t(E, d)$.

5) $\mathbf{\frac{1}{|N|}}$**-min-of-claims-and-deficit lower bounds on losses**, if for each $(E, d) \in C^N$, then $d - \mathcal{R}(E, d) \geq \frac{1}{|N|} t(D - E, d)$.

6) **Equal treatment of equals**, if for each $(E, d) \in C^N$ and each pair $\{i, j\} \in N$, if $d_i = d_j$, then $\mathcal{R}_i(E, d) = \mathcal{R}_j(E, d)$.

7) **Equal treatment of equal groups**, if for each $(E, d) \in C^N$ and each pair $\{N', N''\}$ of subsets of N, if $d(N') = d(N'')$, then $\sum_{i \in N'} \mathcal{R}_i(E, d) = \sum_{i \in N''} \mathcal{R}_i(E, d)$.

8) **Order preservation**, if for each $(E, d) \in C^N$ and each pair $\{i, j\} \subset N$, if $d_i \leq d_j$, then $\mathcal{R}_i(E, d) \leq \mathcal{R}_j(E, d)$ (in awards) and $d_i - \mathcal{R}_i(E, d) \leq d_j - \mathcal{R}_j(E, d)$ (in losses).

9) **Group order preservation**, if for each $(E, d) \in C^N$ and each pair $\{N', N''\}$ of subsets of N, if $d(N') \leq d(N'')$, then $\sum_{i \in N'} \mathcal{R}_i(E, d) \leq \sum_{i \in N''} \mathcal{R}_i(E, d)$ (in awards) and $\sum_{i \in N'} (d_i - \mathcal{R}_i(E, d)) \leq \sum_{i \in N''} (d_i - \mathcal{R}_i(E, d))$ (in losses).

10) **Conditional null compensation**, if for each $(E, d) \in C^N$ and each $i \in N$, if $\sum_{j \in N} \max\{0, d_j - d_i\} \geq E$, then $\mathcal{R}_i(E, d) = 0$.

11) **Conditional full compensation**, if for each $(E, d) \in C^N$ and each $i \in N$, if $\sum_{j \in N} \min\{d_j, d_i\} \leq E$, then $\mathcal{R}_i(E, d) = d_i$.

12) **Linked claim-endowment monotonicity**, if for each $(E, d) \in C^N$, each $i \in N$, and each $\delta > 0$, we have $\mathcal{R}_i(E + \delta, (d_{-i}, d_i + \delta)) - \mathcal{R}_i(E, d) \leq \delta$.

13) **Claim monotonicity**, if for each $(E, d) \in C^N$, each $i \in N$, and each $d'_i \geq d_i$, $\mathcal{R}_i(E, (d_{-i}, d'_i)) \geq \mathcal{R}_i(E, d)$.

14) **Population monotonicity**, if for each pair N, N' of subsets of $\mathcal{N}$ such that $N' \subset N$, and each $(E, d) \in C^N$, we have $\mathcal{R}_{N'}(E, d) \leq \mathcal{R}(E, d_{N'})$.

15) **Linked endowment-population monotonicity**, if for each pair N, N' of subsets of $\mathcal{N}$ such that $N' \subset N$, and each $(E, d) \in C^N$, if $d(N') \geq E$, we have $\mathcal{R}(E, d_{N'}) \leq \mathcal{R}_{N'}(E + d(N \backslash N'), d)$.

16) **Progressivity**, if for each $(E, d) \in C^N$ and each pair $\{i, j\} \subset N$, if $0 < d_i \leq d_j$, we have $\frac{\mathcal{R}_i(E,d)}{d_i} \leq \frac{\mathcal{R}_j(E,d)}{d_j}$.

17) **Regressivity**, if for each $(E,d) \in C^N$ and each pair $\{i,j\} \subset N$, if $0 < d_i \leq d_j$, we have $\frac{\mathcal{R}_i(E,d)}{d_i} \geq \frac{\mathcal{R}_j(E,d)}{d_j}$.

18) **No transfer paradox**, if for each $(E,d) \in C^N$, each pair $\{i,j\} \subset N$, each $d'_i > d_i$, and each $0 \leq d'_j < d_j$, if $d'_i + d'_j = d_i + d_j$, then $\mathcal{R}_i(E,(d'_i,d'_j,d_{N\setminus\{i,j\}})) \geq \mathcal{R}_i(E,d)$ and $\mathcal{R}_j(E,(d'_i,d'_j,d_{N\setminus\{i,j\}})) \leq \mathcal{R}_j(E,d)$.

19) **Bounded impact of claims transfer**, if for each $(E,d) \in C^N$, each pair $\{i,j\} \subset N$, each $d'_i > d_i$, and each $0 \leq d'_j < d_j$, if $d'_i + d'_j = d_i + d_j$, then $\mathcal{R}_i(E,(d'_i,d'_j,d_{N\setminus\{i,j\}})) - \mathcal{R}_i(E,d) \leq d'_i - d_i$ and $\mathcal{R}_j(E,d) - \mathcal{R}_j(E,(d'_i,d'_j,d_{N\setminus\{i,j\}})) \leq d_j - d'_j$.

20) **Concavity**, if for each $(E,d) \in C^N$, each triple $\{E,E',E''\} \subset \mathbb{R}^+$ such that $0 < E < E' < E'' \leq D$, and each pair $\{i,j\} \subset N$, if $0 < d_i \leq d_j$, then $\frac{\mathcal{R}_j(E',d)-\mathcal{R}_j(E,d)}{\mathcal{R}_i(E',d)-\mathcal{R}_i(E,d)} \geq \frac{\mathcal{R}_j(E'',d)-\mathcal{R}_j(E',d)}{\mathcal{R}_i(E'',d)-\mathcal{R}_i(E',d)}$, if these ratios are well defined.

21) **Convexity**, if for each $(E,d) \in C^N$, each triple $\{E,E',E''\} \subset \mathbb{R}^+$ such that $0 < E < E' < E'' \leq D$, and each pair $\{i,j\} \subset N$, if $0 < d_i \leq d_j$, then $\frac{\mathcal{R}_j(E',d)-\mathcal{R}_j(E,d)}{\mathcal{R}_i(E',d)-\mathcal{R}_i(E,d)} \leq \frac{\mathcal{R}_j(E'',d)-\mathcal{R}_j(E',d)}{\mathcal{R}_i(E'',d)-\mathcal{R}_i(E',d)}$, if these ratios are well defined.

22) **Endowment monotonicity**, if for each $(E,d) \in C^N$ and each $E' > E$, if $D \geq E'$ then $\mathcal{R}(E',d) \geq \mathcal{R}(E,d)$.

23) **Order preservation under endowment variations**, if for each $(E,d) \in C^N$, each pair $\{i,j\} \subset N$ and each $E' > E$, if $D \geq E'$ and $d_i \leq d_j$, then $\mathcal{R}_i(E',d) - \mathcal{R}_i(E,d) \leq \mathcal{R}_j(E',d) - \mathcal{R}_j(E,d)$.

24) **Order preservation under claims variations**, if for each $(E,d) \in C^N$ with $|N| \geq 3$, each $i \in N$, each $d'_i > d_i$, and each pair $\{j,k\} \subset N\setminus\{i\}$, if $d_j \leq d_k$, then $\mathcal{R}_j(E,d) - \mathcal{R}_j(E,(d_{-i},d'_i)) \leq \mathcal{R}_k(E,d) - \mathcal{R}_k(E,(d_{-i},d'_i))$.

25) **Minimal rights first**, if for each $(E,d) \in C^N$, $\mathcal{R}(E,d) = m(E,d) + \mathcal{R}\big(E - \sum_{i\in N} m_i(E,d), d - m(E,d)\big)$.

26) **Claims truncation invariance**, if for each $(E,d) \in C^N$, $\mathcal{R}(E,d) = \mathcal{R}(E,t(E,d))$.

27) **Composition down**, if for each $(E,d) \in C^N$ and each $E' < E$, we have $\mathcal{R}(E',d) = \mathcal{R}(E',\mathcal{R}(E,d))$.

28) **Composition up**, if for each $(E,d) \in C^N$ and each $E' > E$ such that $D \geq E'$, we have $\mathcal{R}(E',d) = \mathcal{R}(E,d) + \mathcal{R}(E'-E, d-\mathcal{R}(E,d))$.

29) **Midpoint property**, if for each $(E,d) \in C^N$ such that $E = \frac{1}{2}D$, then $\mathcal{R}(E,d) = \frac{d}{2}$.

30) **Self-duality**, if for each $(E,d) \in C^N$, $\mathcal{R}(E,d) = d - \mathcal{R}\big(D-E,d\big)$.

31) **Claims separability**, if for each pair $(E,d),(E',d') \in C^N$ and each $N' \subset N$, if $d_{N'} = d'_{N'}$, $E = E'$, and $\sum_{i\in N'} \mathcal{R}_i(E,d) = \sum_{i\in N'} \mathcal{R}_i(E',d')$, then $\mathcal{R}_{N'}(E,d) = \mathcal{R}_{N'}(E',d')$.

32) **Claims-and-endowment separability**, if for each pair $(E,d),(E',d') \in C^N$ and each $N' \subset N$, if $d_{N'} = d'_{N'}$ and $\sum_{i\in N'} \mathcal{R}_i(E,d) = \sum_{i\in N'} \mathcal{R}_i(E',d')$, then $\mathcal{R}_{N'}(E,d) = \mathcal{R}_{N'}(E',d')$.

33) **Endowment convexity**, if for each $d \in \mathbb{R}^N_+$, each pair $\{E,E'\} \subset \mathbb{R}_+$ such that $D \geq \max\{E,E'\}$, and each $\lambda \in [0,1]$, $\mathcal{R}(\lambda E + (1-\lambda)E', d) = \lambda\mathcal{R}(E,d) + (1-\lambda)\mathcal{R}(E',d)$.

34) **Claims-and-endowment uniformity**, if for each pair $(E,d),(E',d') \in C^N$, and each $N' \subset N$, if $d_{N'} = d'_{N'}$, then either $\mathcal{R}_{N'}(E,d) \geq \mathcal{R}_{N'}(E',d')$ or $\mathcal{R}_{N'}(E,d) \leq \mathcal{R}_{N'}(E',d')$.

35) **Endowment-and-population uniformity**, if for each pair N, N' of subsets of $\mathcal{N}$ such that $N' \subset N$, each $(E,d) \in C^N$, and each $(E',d') \in C^{N'}$, if $d_{N'} = d'$, then either $\mathcal{R}_{N'}(E,d) \geq \mathcal{R}(E',d')$ or $\mathcal{R}_{N'}(E,d) \leq \mathcal{R}(E',d')$.

36) **No advantageous transfer**, if for each $(E, d) \in C^N$, each $N' \subset N$, and each $(d'_i)_{i\in N'}$, if $d'(N') = d(N')$, then $\sum_{i\in N'} \mathcal{R}_i(E, ((d'_i)_{i\in N'}, d_{N\setminus N'})) = \sum_{i\in N'} \mathcal{R}_i(E, d)$.

37) **Summation independence**, if for each $(E, d) \in C^N$, each $i \in N$, and $N' = N\setminus\{i\}$, for each $(d'_j)_{j\in N'} \in \mathbb{R}_+^{N'}$, if $d'(N') = d(N')$, then $\mathcal{R}_i(E, (d_i, (d'_j)_{j\in N'})) = \mathcal{R}_i(E, d)$.

38) **Consistency**, if for each pair N, N' of subsets of $\mathcal{N}$ such that $N' \subset N$, and each $(E, d) \in C^N$ if $x = \mathcal{R}(E, d)$, then $x_{N'} = \mathcal{R}(x(N'), d_{N'})$.

39) **Bilateral consistency**, if for each pair N, N' of subsets of $\mathcal{N}$ such that $N' \subset N$ and $|N'| = 2$, and each $(E, d) \in C^N$ if $x = \mathcal{R}(E, d)$, then $x_{N'} = \mathcal{R}(x(N'), d_{N'})$.

40) **Converse consistency**, if for each pair N, N' of subsets of $\mathcal{N}$ such that $N' \subset N$ and $|N'| = 2$, each $(E, d) \in C^N$, and each $x \in X(E, d)$, we have $x_{N'} = \mathcal{R}(x(N'), d_{N'})$, then $x = \mathcal{R}(E, d)$.

41) **Null claims consistency**, if for each pair N, N' of subsets of $\mathcal{N}$ such that $N' \subset N$ and each $(E, d) \in C^N$, if $d_{N\setminus N'} = 0$, then $\mathcal{R}_{N'}(E, d) = \mathcal{R}(E, d_{N'})$.

42) **Null compensation consistency**, if for each pair N, N' of subsets of $\mathcal{N}$ such that $N' \subset N$, and each $(E, d) \in C^N$, if $\mathcal{R}_{N\setminus N'}(E, d) = 0$, then $\mathcal{R}_{N'}(E, d) = \mathcal{R}(E, d_{N'})$.

43) **Full compensation consistency**, if for each pair N, N' of subsets of $\mathcal{N}$ such that $N' \subset N$, and each $(E, d) \in C^N$, if $\mathcal{R}_{N\setminus N'}(E, d) = d_{N\setminus N'}$, then $\mathcal{R}_{N'}(E, d) = \mathcal{R}(E - d(N\setminus N'), d_{N'})$.

44) **No advantageous merging**, if for each pair N, N' of subsets of $\mathcal{N}$ such that $N' \subset N$, each $(E, d) \in C^N$, and each $d' \in \mathbb{R}_+^{N'}$, if there is $i \in N'$ such that $d'_i = d_i + d(N\setminus N')$ and for each $k \in N'\setminus\{i\}$, $d'_k = d_k$, then $\mathcal{R}_i(E, d') \leq \mathcal{R}_i(E, d) + \sum_{j\in N\setminus N'} \mathcal{R}_j(E, d)$.

45) **No advantageous splitting**, if for each pair N, N' of subsets of $\mathcal{N}$ such that $N' \subset N$, each $(E, d) \in C^N$, and each $d' \in \mathbb{R}_+^{N'}$, if there is $i \in N'$ such that $d'_i = d_i + d(N\setminus N')$ and for each $k \in N'\setminus\{i\}$, $d'_k = d_k$, then $\mathcal{R}_i(E, d') \geq \mathcal{R}_i(E, d) + \sum_{j\in N\setminus N'} \mathcal{R}_j(E, d)$.

46) **Order preservation under population variations**, if for each pair N, N' of subsets of $\mathcal{N}$ such that $N' \subset N$, each $(E, d) \in C^N$, and each pair $\{i, j\} \subset N'$, if $d(N') \geq E$ and $d_i \leq d_j$, then $\mathcal{R}_i(E, d_{N'}) - \mathcal{R}_i(E, d) \leq \mathcal{R}_j(E, d_{N'}) - \mathcal{R}_j(E, d)$.

47) **Division invariance**, if for each pair N, N' of subsets of $\mathcal{N}$ such that $N' \supset N$, each $(E, d) \in C^N$ and each $k \in \mathbb{N}$. Let (E', d') be a k-replica of (E, d) with associated partition $(N^i)_{i\in N}$, and $\mathcal{R}(E', d')$ be a k-replica of some awards vector $x \in X(E, d)$ associated with the same partition, we have $\mathcal{R}(E, d) = x$.

48) **Replication invariance**, if for each pair N, N' of subsets of $\mathcal{N}$ such that $N' \supset N$, each $(E, d) \in C^N$ and each $k \in \mathbb{N}$. Let (E', d') be a k-replica of (E, d) with associated partition $(N^i)_{i\in N}$, then for each $i \in N$ and each $j \in N^i$ we have $\mathcal{R}_j(E', d') = \mathcal{R}_i(E, d)$.

Value

A table with the rules and the properties. If a rule satisfies a property it returns 1, and 0 otherwise. By default, it returns a table with all rules and properties.

References

Mirás Calvo, M.Á., Núñez Lugilde, I., Quinteiro Sandomingo, C., and Sánchez-Rodríguez, E. (2023). Refining the Lorenz-ranking of rules for claims problems on restricted domains. International Journal of Economic Theory 19(3), 526-558.

Mirás Calvo, M.A., Núñez Lugilde, I., Quinteiro Sandomingo, C., and Sánchez-Rodríguez, E. (2024). An algorithm to compute the average-of-awards rule for claims problems with an application to the allocation of CO_2 emissions. Annals of Operations Research 336, 1435-1459.

Mirás Calvo, M.A., Quinteiro Sandomingo, C., and Sánchez-Rodríguez, E. (2022). The average-of-awards rule for claims problems. Social Choice and Welfare 59, 863-888.

Mirás Calvo,M.Á., Núñez Lugilde, I., Quinteiro Sandomingo, C., and Sánchez Rodríguez,E. (2025). On how the rules that extend the concede-and-divide principle differ for pairs of claimants. Preprint.

Thomson, W. (2019). How to divide when there isn't enough. From Aristotle, the Talmud, and Maimonides to the axiomatics of resource allocation. Cambridge University Press.

Examples

```
Rules=c(AA,Talmud)
Properties=c(1:10)
axioms(Rules,Properties)
axioms() #Table with all the rules and properties implemented.
# The minimal overlap rule does not satisfy linked endowment-population
# monotonicity (Mirás Calvo et al. (2024)):
E=1;d=c(1,2,9,10); d3= d[-3]
MOR=MO(E+d[3],d)
MOR3=MO(E,d3)
MOR3[1]>MOR[1]
# The adjusted proportional rule does not satisfy order preservation under
# population variations (Mirás Calvo et al. (2023)):
E=17; d=c(1,2,3,8,10); d3=d[-1]
APR=APRO(E,d)
APR3=APRO(E,d3)
APR3[1]-APR[1]>APR3[2]-APR[2]
```

`Bankruptcy` *Bankruptcy data*

Description

Bankruptcy data: creditors and claims

Usage

```
Bankruptcy
```

Format

A data frame with 8 rows and 2 variables:

[,1]	`Creditor`	categorical	Creditor name
[,2]	`Claim`	numeric	Claim (millions of euro)

Source

The data were obtained from Fiestras et al (2016)

References

Fiestras-Janeiro, M.G., Sánchez-Rodríguez, E., and Schuster, M. (2016). A precedence constraint value revisited. Top, 25, 156–179

Examples

```
data(Bankruptcy)
Bankruptcy
E= 230
allrules(E,Bankruptcy$Claim)
```

CD *Concede-and-divide rule*

Description

This function returns the awards vector assigned by the concede-and-divide (CD) rule to a two-claimant problem.

Usage

```
CD(E, d, name = FALSE)
```

Arguments

E	The endowment.
d	The vector of two claims.
name	A logical value.

Details

Let $E \geq 0$ be the endowment to be divided and $d = (d_1, d_2) \in \mathbb{R}^2_+$ the vector of claims such that $d_1 + d_2 \geq E$.

The concede-and-divide rule (CD) first assigns to each of the two claimants the difference between the endowment and the other agent's claim (or 0 if this difference is negative), and divides the remainder equally. That is, for each $i \in \{1, 2\}$,

$$\mathrm{CD}_i(E, d) = \max\{E - d_j, 0\} + \frac{E - \max\{E - d_i, 0\} - \max\{E - d_j, 0\}}{2}.$$

Several rules are extensions of the concede-and-divide rule to general populations: AA, APRO, MO, RA, and Talmud.

Value

The awards vector selected by the CD rule. If `name = TRUE`, the name of the function (CD) as a character string.

References

Aumann, R. and Maschler, M., (1985). Game theoretic analysis of a bankruptcy problem from the Talmud. Journal of Economic Theory 36, 195–213.

Mirás Calvo,M.Á., Núñez Lugilde, I., Quinteiro Sandomingo, C., and Sánchez Rodríguez,E. (2025). On how the rules that extend the concede-and-divide principle differ for pairs of claimants. Preprint.

Thomson, W. (2019). How to divide when there isn't enough. From Aristotle, the Talmud, and Maimonides to the axiomatics of resource allocation. Cambridge University Press.

See Also

allrules, AA, APRO, MO, pathawards, RA, Talmud.

Examples

```
E=10
d=c(7,8)
CD(E,d)
# Talmud, RA, MO, APRO, and AA coincide with CD for two-claimant problems
Talmud(E,d)
RA(E,d)
MO(E,d)
APRO(E,d)
AA(E,d)
```

CE *Constrained egalitarian rule*

Description

This function returns the awards vector assigned by the constrained egalitarian rule (CE) rule to a claims problem.

Usage

```
CE(E, d, name = FALSE)
```

Arguments

`E`	The endowment.
`d`	The vector of claims.
`name`	A logical value.

Details

Let $N = \{1, \ldots, n\}$ be the set of claimants, $E \geq 0$ the endowment to be divided and $d \in \mathbb{R}_+^N$ the vector of claims such that $D = \sum_{i \in N} d_i \geq E$.

Rearrange the claims from small to large, $0 \leq d_1 \leq ... \leq d_n$. The constrained egalitarian rule (CE) coincides with the constrained equal awards rule (CEA) applied to the problem $(E, d/2)$ if the endowment is less or equal than the half-sum of the claims, $D/2$. Otherwise, any additional unit is assigned to claimant 1 until she/he receives the minimum of the claim and half of d_2. If this minimun is d_1, she/he stops there. If it is not, the next increment is divided equally between claimants 1 and 2 until claimant 1 receives d_1 (in this case she drops out) or they reach $d_3/2$. If claimant 1 leaves, claimant 2 receives any additional increment until she/he reaches d_2 or $d_3/2$. In the case that claimant 1 and 2 reach $d_3/2$, any additional unit is divided between claimants 1, 2, and 3 until the first one receives d_1 or they reach $d_4/2$, and so on. Therefore, for each $i \in N$,

$$\text{CE}_i(E,d) = \begin{cases} \min\{\frac{d_i}{2}, \lambda\} & \text{if } E \leq \frac{1}{2}D \\ \max\{\frac{d_i}{2}, \min\{d_i, \lambda\}\} & \text{if } E \geq \frac{1}{2}D \end{cases},$$

where $\lambda \geq 0$ is chosen such that $\sum\limits_{i \in N} \text{CE}_i(E,d) = E$.

Value

The awards vector selected by the CE rule. If `name = TRUE`, the name of the function (CE) as a character string.

References

Chun, Y., Schummer, J., Thomson, W. (2001). Constrained egalitarianism: a new solution for claims problems. Seoul Journal of Economics 14, 269–297.

Thomson, W. (2019). How to divide when there isn't enough. From Aristotle, the Talmud, and Maimonides to the axiomatics of resource allocation. Cambridge University Press.

See Also

allrules, axioms, CEA, PIN, Talmud.

Examples

```
E=10
d=c(2,4,7,8)
CE(E,d)
```

CEA — *Constrained equal awards rule*

Description

This function returns the awards vector assigned by the constrained equal awards rule (CEA) to a claims problem.

Usage

```
CEA(E, d, name = FALSE)
```

Arguments

E	The endowment.
d	The vector of claims.
name	A logical value.

Details

Let $N = \{1, \ldots, n\}$ be the set of claimants, $E \geq 0$ the endowment to be divided and $d \in \mathbb{R}_+^N$ the vector of claims such that $\sum_{i \in N} d_i \geq E$.

The constrained equal awards rule (CEA) equalizes awards under the constraint that no individual's award exceeds his/her claim. Then, claimant i receives the minimum of the claim and a value $\lambda \geq 0$ chosen so as to achieve balance. That is, for each $i \in N$,

$$\mathrm{CEA}_i(E, d) = \min\{d_i, \lambda\},$$

where $\lambda \geq 0$ is chosen such that $\sum_{i \in N} \mathrm{CEA}_i(E, d) = E$.

The constrained equal awards rule corresponds to the Dutta-Ray solution to the associated (pessimistic) coalitional game. The CEA and CEL rules are dual.

Value

The awards vector selected by the CEA rule. If `name = TRUE`, the name of the function (CEA) as a character string.

References

Maimonides, Moses, [1135-1204], Book of Judgements (translated by Rabbi Elihahu Touger, 2000), New York and Jerusalem: Moznaim Publishing Corporation, 2000.

Thomson, W. (2019). How to divide when there isn't enough. From Aristotle, the Talmud, and Maimonides to the axiomatics of resource allocation. Cambridge University Press.

See Also

allrules, axioms, CE, CEL, AV, PIN, Talmud, RTalmud.

Examples

```
E=10
d=c(2,4,7,8)
CEA(E,d)
# CEA and CEL are dual: CEA(E,d)=d-CEL(D-E,d)
D=sum(d)
d-CEL(D-E,d)
```

`CEL` *Constrained equal losses rule*

Description

This function returns the awards vector assigned by the constrained equal losses rule (CEL) to a claims problem.

Usage

```
CEL(E, d, name = FALSE)
```

Arguments

`E`	The endowment.
`d`	The vector of claims.
`name`	A logical value.

Details

Let $N = \{1, \ldots, n\}$ be the set of claimants, $E \geq 0$ the endowment to be divided and $d \in \mathbb{R}_+^N$ the vector of claims such that $\sum_{i \in N} d_i \geq E$.

The constrained equal losses rule (CEL) equalizes losses under the constraint that no award is negative. Then, claimant i receives the maximum of zero and the claim minus a number $\lambda \geq 0$ chosen so as to achieve balance. That is, for each $i \in N$,

$$\text{CEL}_i(E, d) = \max\{0, d_i - \lambda\},$$

where $\lambda \geq 0$ is chosen such that $\sum_{i \in N} \text{CEL}_i(E, d) = E$.

CEA and CEL are dual rules.

Value

The awards vector selected by the CEL rule. If `name = TRUE`, the name of the function (CEL) as a character string.

References

Maimonides, Moses, [1135-1204], Book of Judgements (translated by Rabbi Elihahu Touger, 2000), New York and Jerusalem: Moznaim Publishing Corporation, 2000.

Thomson, W. (2019). How to divide when there isn't enough. From Aristotle, the Talmud, and Maimonides to the axiomatics of resource allocation. Cambridge University Press.

See Also

allrules, axioms, CE, CEA, AV, PIN, Talmud, RTalmud.

Examples

```
E=10
d=c(2,4,7,8)
CEL(E,d)
# CEL and CEA are dual: CEL(E,d)=d-CEA(D-E,d)
D=sum(d)
d-CEA(D-E,d)
```

CO2emissions *CO2 emissions (kt) data*

Description

CO2 emissions (kt) data from different countries and regions in 2014

Usage

```
CO2emissions
```

Format

A data frame with 20 rows and 2 variables:

[,1]	`Region`	categorical	Country or Region name
[,2]	`Emissions`	numeric	CO2 emissions (kt)

Source

The data were obtained from Climate Change Data, World Bank Group https://climateknowledgeportal.worldbank.org/

References

Mirás Calvo, M.A., Núñez Lugilde, I., Quinteiro Sandomingo, C., and Sánchez-Rodríguez, E. (2024). An algorithm to compute the average-of-awards rule for claims problems with an application to the allocation of CO_2 emissions. Annals of Operations Research 336, 1435-1459.

Examples

```
data(CO2emissions)
head(CO2emissions)
E=31284288 #Emissions for 2015
allrules(E,CO2emissions$Emissions)
par(mfrow = c(2, 3))
E0 <- 33857455
Rules=c(Talmud,CEA,CEL,PRO)
percentage = 0.076
times = 20
for (claimant in 1:6) {
dynamicplot(E0, CO2emissions$Emissions, Rules, claimant, percentage, times, legend=FALSE)
}
```

---

coalitionalgame *Coalitional game associated with a claims problem*

Description

This function returns the pessimistic and optimistic coalitional games associated with a claims problem.

Usage

```
coalitionalgame(E, d, opt = FALSE, lex = FALSE)
```

Arguments

E	The endowment.
d	The vector of claims.
opt	Logical parameter. If opt = TRUE, both the pessimist and optimistic associated coalitional games are given. By default, opt = FALSE, and only the associated pessimistic coalitional game is computed.
lex	Logical parameter. If lex = TRUE, coalitions of claimants are ordered lexicographically. By default, lex = FALSE, and coalitions are ordered using their binary representations.

Details

Let $N = \{1, \ldots, n\}$ be the set of claimants, $E \geq 0$ the endowment to be divided and $d \in \mathbb{R}_+^N$ the vector of claims such that $\sum_{i \in N} d_i \geq E$. For each coalition $S \in 2^N$, let $d(S) = \sum_{j \in S} d_j$ and $N \backslash S$ be the complementary coalition of S.

Given a claims problem (E, d), its associated pessimistic coalitional game is the game $v_{pes} : 2^N \rightarrow \mathbb{R}$ assigning to each coalition $S \in 2^N$,

$$v_{pes}(S) = \max\{0, E - d(N \backslash S)\}.$$

Given a claims problem (E, d), its associated optimistic coalitional game is the game $v_{opt} : 2^N \rightarrow \mathbb{R}$ assigning to each coalition $S \in 2^N$,

$$v_{opt}(S) = \min\{E, d(S)\}.$$

The optimistic and the pessimistic coalitional games are dual games, that is, for all $S \in 2^N$,

$$v_{opt}(S) = E - v_{pes}(N \backslash S).$$

An efficient way to represent a nonempty coalition $S \in 2^N$ is by identifying it with the binary sequence $a_n a_{n-1} \dots a_1$, where $a_i = 1$ if $i \in S$ and $a_i = 0$ otherwise. Therefore, each coalition S is represented by the number associated with its binary representation: $\sum_{i \in S} 2^{i-1}$. Then coalitions can be ordered by their associated numbers.

Alternatively, coalitions can be ordered lexicographically.

Given a claims problem (E, d), its associated coalitional game v can be represented by the vector whose coordinates are the values assigned by v to all the nonempty coalitions. For instance. if $n = 3$, the associated coalitional game can be represented by the vector of the values of all the 7 nonempty coalitions, ordered using the binary representation:

$$v = [v(\{1\}), v(\{2\}), v(\{1,2\}), v(\{3\}), v(\{1,3\}), v(\{2,3\}), v(\{1,2,3\})].$$

Alternatively, the coordinates can be ordered lexicographically:

$$v = [v(\{1\}), v(\{2\}), v(\{3\}), v(\{1,2\}), v(\{1,3\}), v(\{2,3\}), v(\{1,2,3\})].$$

When $n = 4$, the associated coalitional game can be represented by the vector of the values of all the 15 nonempty coalitions, ordered using the binary representation:

$v = [v(\{1\}), v(\{2\}), v(\{1,2\}), v(\{3\}), v(\{1,3\}), v(\{2,3\}), v(\{1,2,3\}), v(\{4\}), \dots$

$\dots, v(\{1,4\}), v(\{2,4\}), v(\{1,2,4\}), v(\{3,4\}), v(\{1,3,4\}), v(\{2,3,4\}), v(\{1,2,3,4\})].$

Alternatively, the coordinates can be ordered lexicographically:

$v = [v(\{1\}), v(\{2\}), v(\{3\}), v(\{4\}), v(\{1,2\}), v(\{1,3\}), v(\{1,4\}), v(\{2,3\}), \dots$

$\dots v(\{2,4\}), v(\{3,4\}), v(\{1,2,3\}), v(\{1,2,4\}), v(\{1,3,4\}), v(\{2,3,4\}), v(\{1,2,3,4\})].$

Value

The pessimistic (and optimistic) associated coalitional game(s).

References

O'Neill, B. (1982) A problem of rights arbitration from the Talmud. Mathematical Social Sciences 2, 345–371.

See Also

setofawards.

Examples

```
E=10
d=c(2,4,7,8)
v=coalitionalgame(E,d,opt=TRUE,lex=TRUE)
#The pessimistic and optimistic coalitional games are dual games
v_pes=v$v_pessimistic_lex
v_opt=v$v_optimistic_lex
v_opt[1:14]==10-v_pes[14:1]
```

---

cumawardscurve *Cumulative awards curve*

Description

The graphical representation of the cumulative curves of a rule (or several rules) with respect to a given rule, for a claims problem.

Usage

```
cumawardscurve(E, d, Rule = PRO, Rules, col = NULL, legend = TRUE)
```

Arguments

`E`	The endowment.
`d`	The vector of claims.
`Rule`	Principal Rule: AA, APRO, CE, CEA, CEL, AV, DT, MO, PIN, PRO, RA, Talmud, RTalmud. By default, `Rule = PRO`.
`Rules`	The rules: AA, APRO, CE, CEA, CEL, AV, DT, MO, PIN, PRO, RA, Talmud, RTalmud.
`col`	The colours. If `col = NULL` then the sequence of default colours is: c("red", "blue", "green", "yellow", "pink","orange", "coral4", "darkgray", "burlywood3", "black", "darkorange", "darkviolet").
`legend`	A logical value. The colour legend is shown if `legend = TRUE`.

Details

Let $N = \{1, \ldots, n\}$ be the set of claimants, $E \geq 0$ the endowment to be divided and $d \in \mathbb{R}_+^N$ the vector of claims such that $\sum_{i \in N} d_i \geq E$.

Rearrange the claims from small to large, $0 \leq d_1 \leq ... \leq d_n$. The cumulative curve allows us to compare the division recommended by a specific rule $\mathcal{R}$ with the division the division recommended by another specific rule $\mathcal{S}$.

The cumulative awards curve of a rule $\mathcal{S}$ with respect of a rule $\mathcal{R}$ for the claims problem (E, d) is the polygonal path connecting the $n + 1$ points

$$(0,0), \Big(\frac{\mathcal{R}_1}{E}, \frac{\mathcal{S}_1}{E}\Big), \ldots, \Big(\frac{\sum_{i=1}^{n-1} \mathcal{R}_i}{E}, \frac{\sum_{i=1}^{n-1} \mathcal{S}_i}{E}\Big), (1,1).$$

The cumulative awards curve fully captures the Lorenz ranking of rules: if a rule $\mathcal{R}$ Lorenz-dominates a rule $\mathcal{S}$ then, for each claims problem, the cumulative curve of $\mathcal{R}$ lies above the cumulative curve of $\mathcal{S}$. If $\mathcal{R} =$ PRO, the cumulative curve coincides with the cumulative claims-awards curve.

Value

The graphical representation of the cumulative curves of a rule (or several rules) for a claims problem.

References

Lorenz, M. O. (1905). Methods of measuring the concentration of wealth. Publications of the American statistical association 9(70), 209-219.

Mirás Calvo, M.Á., Núñez Lugilde, I., Quinteiro Sandomingo, C., and Sánchez Rodríguez, E. (2023). Deviation from proportionality and Lorenz-domination for claims problems. Review of Economic Design 27, 439-467.

See Also

allrules, deviationindex, giniindex, indexgpath, lorenzcurve, lorenzdominance.

Examples

```
E=10
d=c(2,4,7,8)
Rule=PRO
Rules=c(AA,RA,Talmud,CEA,CEL)
cumawardscurve(E,d,Rule,Rules)
```

`deviationindex` *Deviation index*

Description

This function returns the deviation index and the signed deviation index for a rule with respect to another rule.

Usage

```
deviationindex(E, d, R, S)
```

Arguments

E	The endowment.
d	The vector of claims.
R	A rule : AA, APRO, CE, CEA, CEL, AV, DT, MO, PIN, PRO, RA, Talmud, RTalmud.
S	A rule: AA, APRO, CE, CEA, CEL, AV, DT, MO, PIN, PRO, RA, Talmud, RTalmud.

Details

Let $N = \{1, \ldots, n\}$ be the set of claimants, $E \geq 0$ the endowment to be divided and $d \in \mathbb{R}_+^N$ the vector of claims such that $\sum_{i \in N} d_i \geq E$.

Rearrange the claims from small to large, $0 \leq d_1 \leq ... \leq d_n$. The signed deviation index of the rule $\mathcal{S}$ with respect to the rule $\mathcal{R}$ for the problem (E, d), denoted by $I(\mathcal{R}(E, d), \mathcal{S}(E, d))$, is the ratio of the area that lies between the identity line and the cumulative curve over the total area under the identity line.

Let $\mathcal{R}_0 = 0$ and $\mathcal{S}_0 = 0$. For each $k = 1, \ldots, n$ define $X_k = \frac{1}{E} \sum_{j=0}^{k} \mathcal{R}_j$ and $Y_k = \frac{1}{E} \sum_{j=0}^{k} \mathcal{S}_j$. Then,

$$I(\mathcal{R}(E, d), \mathcal{S}(E, d)) = 1 - \sum_{k=1}^{n} \Big(X_k - X_{k-1}\Big)\Big(Y_k + Y_{k-1}\Big).$$

In general $-1 \leq I(\mathcal{R}(E, d), \mathcal{S}(E, d)) \leq 1$.

The deviation index of the rule $\mathcal{S}$ with respect to the rule $\mathcal{R}$ for the problem (E, d), denoted by $I^+(\mathcal{R}(E, d), \mathcal{S}(E, d))$, is the ratio of the area between the line of the cumulative sum of the distribution proposed by the rule $\mathcal{R}$ and the cumulative curve over the area under the line $x = y$.

In general $0 \leq I^+(\mathcal{R}(E, d), \mathcal{S}(E, d)) \leq 1$.

The proportionality deviation index is the deviation index when $\mathcal{R} =$ PRO. The proportionality deviation index of the proportional rule is zero for all claims problems. The signed proportionality deviation index is the signed deviation index with $\mathcal{R} =$ PRO.

Value

The deviation index and the signed deviation index of a rule for a claims problem.

References

Ceriani, L. and Verme, P. (2012). The origins of the Gini index: extracts from Variabilitá e Mutabilitá (1912) by Corrado Gini. The Journal of Economic Inequality 10(3), 421-443.

Mirás Calvo, M.Á., Núñez Lugilde, I., Quinteiro Sandomingo, C., and Sánchez Rodríguez, E. (2023). Deviation from proportionality and Lorenz-domination for claims problems. Review of Economic Design 27, 439-467.

See Also

allrules, cumawardscurve, giniindex, indexgpath, lorenzcurve, lorenzdominance.

Examples

```
E=10
d=c(2,4,7,8)
R=CEA
S=AA
deviationindex(E,d,R,S)
#The deviation index of rule R with respect of the rule R is 0.
deviationindex(E,d,PRO,PRO)
```

DT *Dominguez-Thomson rule*

Description

This function returns the awards vector assigned by the Dominguez-Thomson rule (DT) to a claims problem.

Usage

```
DT(E, d, name = FALSE)
```

Arguments

E	The endowment.
d	The vector of claims.
name	A logical value.

Details

Let $N = \{1, \ldots, n\}$ be the set of claimants, $E \geq 0$ the endowment to be divided and $d \in \mathbb{R}_+^N$ the vector of claims such that $\sum_{i \in N} d_i \geq E$.

The truncated claim of claimant $i \in N$ in (E, d) is the minimum of the claim and the endowment:

$$t_i(E, d) = \min\{d_i, E\},\ i = 1, \ldots, n.$$

Let $t(E, d) = (t_1(E, d), \ldots, t_n(E, d))$ be the vector of truncated claims and $b(E, d) = \frac{1}{n} t(E, d)$.

Let $(E^1, d^1) = (E, d)$. For each $k \geq 2$ define:

$$(E^k, d^k) = \Big(E^{k-1} - \sum_{i=1}^{n} b_i(E^{k-1}, d^{k-1}), d^{k-1} - b(E^{k-1}, d^{k-1})\Big).$$

In step 1, the endowment is E and the claims vector is d. For $k \geq 2$, the endowment is the remainder once all the claimants have received the amount of the previous steps and the new vector of claims is readjusted accordingly. Observe that neither the endowment nor each agent's claim ever increases from one step to the next. This recursive process exhausts E in the limit.

For each (E, d), the Dominguez-Thomson rule (DT) assigns the awards vector:

$$\text{DT}(E, d) = \sum_{k=1}^{\infty} b(E^k, d^k).$$

Value

The awards vector selected by the DT rule. If `name = TRUE`, the name of the function (DT) as a character string.

References

Domínguez, D. and Thomson, W. (2006). A new solution to the problem of adjudicating conflicting claims. Economic Theory 28(2), 283-307.

Thomson, W. (2019). How to divide when there isn't enough. From Aristotle, the Talmud, and Maimonides to the axiomatics of resource allocation. Cambridge University Press.

See Also

allrules.

Examples

```
E=10
d=c(2,4,7,8)
DT(E,d)
```

---

dynamicplot *Dynamic plot*

Description

For each claimaint, it plots the awards of the chosen rules for a dynamic model with t periods.

Usage

```
dynamicplot(
  E,
  d,
  Rules,
  claimant,
  percentage,
  times,
  col = NULL,
  legend = TRUE
)
```

Arguments

E	The endowment.
d	The vector of claims
Rules	The rules: AA, APRO, CE, CEA, CEL, AV, DT, MO, PIN, PRO, RA, Talmud, RTalmud.

`claimant`	A claimant.
`percentage`	A number in (0,1).
`times`	Number of iterations.
`col`	The colours. If `col=NULL` then the sequence of default colours is: c("red", "blue", "green", "yellow", "pink", "orange", "coral4", "darkgray", "burlywood3", "black", "darkorange", "darkviolet").
`legend`	A logical value. The colour legend is shown if `legend=TRUE`.

Details

Let $N = \{1, \ldots, n\}$ be the set of claimants, $E \geq 0$ the endowment to be divided and $d \in \mathbb{R}_+^N$ the vector of claims such that $\sum_{i \in N} d_i \geq E$.

A vector $x = (x_1, \ldots, x_n)$ is an awards vector for the claims problem (E, d) if $0 \leq x \leq d$ and satisfies the balance requirement, that is, $\sum_{i=1}^{n} x_i = E$.

A rule is a function that assigns to each claims problem (E, d) an awards vector, that is, a division between the claimants of the amount available.

The formal definitions of the main rules are given in the corresponding function help.

Given l a natural number, the function solves each claims problem in time t, which is (E_t, d), with $E_t = (1-p)^t E$, $p \in (0, 1)$ and $t = 1, \ldots, l$.

Value

This function represents the awards proposed by different rules for a claimant if the resource decreases in each iteration by a given percentage.

References

Mirás Calvo, M.A., Núñez Lugilde, I., Quinteiro Sandomingo, C., and Sánchez-Rodríguez, E. (2023). An algorithm to compute the average-of-awards rule for claims problems with an application to the allocation of CO_2 emissions. Annals of Operations Research 336, 1435-1459.

Thomson, W. (2019). How to divide when there isn't enough. From Aristotle, the Talmud, and Maimonides to the axiomatics of resource allocation. Cambridge University Press.

See Also

allrules, pathawards, pathawards3, schedrule, schedrules.

Examples

```
E=10
d=c(2,4,7,8)
Rules=c(Talmud,RA,AA,PRO)
claimant=1
percentage=0.076
times=10
dynamicplot(E,d,Rules,claimant,percentage,times)
```

giniindex *Gini index*

Description

This function returns the Gini index of any rule for a claims problem.

Usage

```
giniindex(E, d, Rule)
```

Arguments

E	The endowment.
d	The vector of claims.
Rule	A rule: AA, APRO, CE, CEA, CEL, AV, DT, MO, PIN, PRO, RA, Talmud, RTalmud.

Details

Let $N = \{1, \ldots, n\}$ be the set of claimants, $E \geq 0$ the endowment to be divided and $d \in \mathbb{R}_+^N$ the vector of claims such that $\sum_{i \in N} d_i \geq E$.

Rearrange the claims from small to large, $0 \leq d_1 \leq ... \leq d_n$. The Gini index is a number aimed at measuring the degree of inequality in a distribution. The Gini index of the rule $\mathcal{R}$ for the problem (E, d), denoted by $G(\mathcal{R}, E, d)$, is the ratio of the area that lies between the identity line and the Lorenz curve of the rule over the total area under the identity line.

Let $\mathcal{R}_0(E, d) = 0$. For each $k = 0, \ldots, n$ define $X_k = \frac{k}{n}$ and $Y_k = \frac{1}{E} \sum_{j=0}^{k} \mathcal{R}_j(E, d)$. Then,

$$G(\mathcal{R}, E, d) = 1 - \sum_{k=1}^{n} \Big(X_k - X_{k-1}\Big)\Big(Y_k + Y_{k-1}\Big).$$

In general $0 \leq G(\mathcal{R}, E, d) \leq 1$.

Value

The Gini index of a rule for a claims problem and the Gini index of the vector of claims.

References

Ceriani, L. and Verme, P. (2012). The origins of the Gini index: extracts from Variabilitá e Mutabilitá (1912) by Corrado Gini. The Journal of Economic Inequality 10(3), 421-443.

Mirás Calvo, M.Á., Núñez Lugilde, I., Quinteiro Sandomingo, C., and Sánchez Rodríguez, E. (2023). Deviation from proportionality and Lorenz-domination for claims problems. Review of Economic Design 27, 439-467.

See Also

cumawardscurve, deviationindex, indexgpath, lorenzcurve, lorenzdominance.

Examples

```
E=10
d=c(2,4,7,8)
Rule=AA
giniindex(E,d,Rule)
# The Gini index of the proportional awards coincides with the Gini index of the vector of claims
giniindex(E,d,PRO)
```

`indexgpath` *Index path*

Description

The function returns the deviation index path or the signed deviation index path for a rule with respect to another rule for a vector of claims.

Usage

```
indexgpath(
  d,
  Rule = PRO,
  Rules,
  signed = TRUE,
  col = NULL,
  points = 201,
  legend = TRUE
)
```

Arguments

`d`	The vector of claims.
`Rule`	Principal Rule: AA, APRO, CE, CEA, CEL, AV, DT, MO, PIN, PRO, RA, Talmud, RTalmud. By default, `Rule = PRO`.
`Rules`	The rules: AA, APRO, CE, CEA, CEL, AV, DT, MO, PIN, PRO, RA, Talmud, RTalmud.
`signed`	A logical value. If `signed = FALSE`, it draws the deviation index path and, if `signed = TRUE` it draws the signed deviation index path. By default, `signed = TRUE`.
`col`	The colours. If `col = NULL` then the sequence of default colours is: c("red", "blue", "green", "yellow", "pink", "orange", "coral4", "darkgray", "burlywood3", "black", "darkorange", "darkviolet").
`points`	The number of endowment values to be drawn.
`legend`	A logical value. The legend is shown if `legend = TRUE`.

Details

Let $N = \{1, \ldots, n\}$ be the set of claimants, $d \in \mathbb{R}^N$ a vector of claims rearranged from small to large, $0 \leq d_1 \leq ... \leq d_n$ and $D = \sum_{i \in N} d_i$.

Given two rules $\mathcal{R}$ and $\mathcal{S}$, consider the function J that assigns to each $E \in (0, D]$ the value $J(E) = I\Big(\mathcal{R}(E, d), \mathcal{S}(E, d)\Big)$, that is, the signed deviation index of the rules $\mathcal{R}$ and $\mathcal{S}$ for the problem (E, d). The graph of J is the signed index path of $\mathcal{S}$ in function of the rule $\mathcal{R}$ for the vector of claims d.

Given two rules $\mathcal{R}$ and $\mathcal{S}$, consider the function J^+ that assigns to each $E \in (0, D]$ the value $J^+(E) = I^+\Big(\mathcal{R}(E, d), \mathcal{S}(E, d)\Big)$, that is, the deviation index of the rules $\mathcal{R}$ and $\mathcal{S}$ for the problem (E, d). The graph of J^+ is the index path of $\mathcal{S}$ in function of the rule $\mathcal{R}$ for the vector of claims d.

The signed index path and the index path are simple tools to visualize the discrepancy of the divisions recommended by a rule for a vector of claims with respect to the divisions recommended by another rule. If $\mathcal{R} =$ PRO, the function draws the proportionality deviation index path or the signed proportionality deviation index path.

Value

This function returns the deviation index path of a rule (or several rules) for a vector of claims.

References

Ceriani, L. and Verme, P. (2012). The origins of the Gini index: extracts from Variabilitá e Mutabilitá (1912) by Corrado Gini. The Journal of Economic Inequality 10(3), 421-443.

Mirás Calvo, M.Á., Núñez Lugilde, I., Quinteiro Sandomingo, C., and Sánchez Rodríguez, E. (2023). Deviation from proportionality and Lorenz-domination for claims problems. Review of Economic Design 27, 439-467.

Thomson, W. (2019). How to divide when there isn't enough. From Aristotle, the Talmud, and Maimonides to the axiomatics of resource allocation. Cambridge University Press.

See Also

allrules, cumawardscurve, deviationindex, giniindex, lorenzcurve, lorenzdominance.

Examples

```
d=c(2,4,7,8)
Rule=PRO
Rules=c(Talmud,RA,AA)
col=c("red","green","blue")
indexgpath(d,Rule,Rules,signed=TRUE,col)
```

`lorenzcurve` *The Lorenz curve*

Description

This function returns the Lorenz curve of any rule for a claims problem.

Usage

```
lorenzcurve(E, d, Rules, col = NULL, legend = TRUE)
```

Arguments

`E`	The endowment.
`d`	The vector of claims.
`Rules`	The rules: AA, APRO, CE, CEA, AV, DT, MO, PIN, PRO, RA, Talmud, RTalmud.
`col`	The colours. If `col=NULL` then the sequence of default colors is: c("red", "blue", "green", "yellow", "pink", "orange", "coral4", "darkgray", "burlywood3", "black", "darkorange", "darkviolet").
`legend`	A logical value. The colour legend is shown if `legend=TRUE`.

Details

Let $N = \{1, \ldots, n\}$ be the set of claimants, $E \geq 0$ the endowment to be divided and $d \in \mathbb{R}_+^N$ the vector of claims such that $\sum_{i \in N} d_i \geq E$.

Rearrange the claims from small to large, $0 \leq d_1 \leq ... \leq d_n$. The Lorenz curve represents the proportion of the awards given to each subset of claimants by a specific rule $\mathcal{R}$ as a function of the cumulative distribution of population.

The Lorenz curve of a rule $\mathcal{R}$ for the claims problem (E, d) is the polygonal path connecting the $n + 1$ points,

$$(0,0), \Big(\frac{1}{n}, \frac{\mathcal{R}_1(E,d)}{E}\Big), \ldots, \Big(\frac{n-1}{n}, \frac{\sum_{i=1}^{n-1} \mathcal{R}_i(E,d)}{E}\Big), (1,1).$$

Basically, it represents the cumulative percentage of the endowment assigned by the rule to each cumulative percentage of claimants.

Value

The graphical representation of the Lorenz curve of a rule (or several rules) for a claims problem.

References

Lorenz, M. O. (1905). Methods of measuring the concentration of wealth. Publications of the American statistical association 9(70), 209-219.

Mirás Calvo, M.Á., Núñez Lugilde, I., Quinteiro Sandomingo, C., and Sánchez Rodríguez, E. (2023a). Deviation from proportionality and Lorenz-domination for claims problems. Review of Economic Design 27, 439-467.

Mirás Calvo, M.Á., Núñez Lugilde, I., Quinteiro Sandomingo, C., and Sánchez-Rodríguez, E. (2023b). Refining the Lorenz-ranking of rules for claims problems on restricted domains. International Journal of Economic Theory 19(3), 526-558.

See Also

cumawardscurve, deviationindex, giniindex, indexgpath, lorenzdominance.

Examples

```
E=10
d=c(2,4,7,8)
Rules=c(AA,RA,Talmud,CEA,CEL)
col=c("red","blue","green","yellow","pink")
lorenzcurve(E,d,Rules,col)
```

---

lorenzdominance — *Lorenz-dominance relation*

Description

This function checks whether or not the awards assigned by two rules to a claims problem are Lorenz-comparable.

Usage

```
lorenzdominance(E, d, Rules, Info = FALSE)
```

Arguments

E	The endowment.
d	The vector of claims.
Rules	The two rules: AA, APRO, CE, CEA, CEL, AV, DT, MO, PIN, PRO, RA, Talmud, RTalmud.
Info	A logical value.

Details

Let $N = \{1, \ldots, n\}$ be the set of claimants, $E \geq 0$ the endowment to be divided and $d \in \mathbb{R}^N_+$ the vector of claims such that $\sum_{i \in N} d_i \geq E$.

A vector $x = (x_1, \ldots, x_n)$ is an awards vector for the claims problem (E, d) if $0 \leq x \leq d$ and satisfies the balance requirement, that is, $\sum_{i=1}^n x_i = E$. A rule is a function that assigns to each claims problem (E, d) an awards vector.

Given a claims problem (E, d), in order to compare a pair of awards vectors $x, y \in X(E, d)$ with the Lorenz criterion, first one has to rearrange the coordinates of each allocation in a non-decreasing order. Then we say that x Lorenz-dominates y (or, that y is Lorenz-dominated by x) if all the cumulative sums of the rearranged coordinates are greater with x than with y. That is, x Lorenz-dominates y if for each $k = 1, \ldots, n-1$,

$$\sum_{j=1}^k x_j \geq \sum_{j=1}^k y_j.$$

Let $\mathcal{R}$ and $\mathcal{S}$ be two rules, we say that $\mathcal{R}$ Lorenz-dominates $\mathcal{S}$ if $\mathcal{R}(E, d)$ Lorenz-dominates $\mathcal{S}(E, d)$ for all (E, d).

Value

If `Info = FALSE`, the Lorenz-dominance relation between the awards vectors selected by both rules. If both awards vectors are equal then `cod = 2`. If the awards vectors are not Lorenz-comparable then `cod = 0`. If the awards vector selected by the first rule Lorenz-dominates the awards vector selected by the second rule then `cod = 1`; otherwise `cod = -1`. If `Info = TRUE`, it also gives the corresponding cumulative sums.

References

Lorenz, M. O. (1905). Methods of measuring the concentration of wealth. Publications of the American statistical association 9(70), 209-219.

Mirás Calvo, M.Á., Núñez Lugilde, I., Quinteiro Sandomingo, C., and Sánchez Rodríguez, E. (2023a). Deviation from proportionality and Lorenz-domination for claims problems. Review of Economic Design 27, 439-467.

Mirás Calvo, M.Á., Núñez Lugilde, I., Quinteiro Sandomingo, C., and Sánchez-Rodríguez, E. (2023b). Refining the Lorenz-ranking of rules for claims problems on restricted domains. International Journal of Economic Theory 19(3), 526-558

See Also

cumawardscurve, deviationindex, giniindex, indexgpath, lorenzcurve.

Examples

```
E=10
d=c(2,4,7,8)
Rules=c(AA,CEA)
lorenzdominance(E,d,Rules)
```

MO *Minimal overlap rule*

Description

This function returns the awards vector assigned by the minimal overlap rule rule (MO) to a claims problem.

Usage

```
MO(E, d, name = FALSE)
```

Arguments

`E`	The endowment.
`d`	The vector of claims.
`name`	A logical value.

Details

Let $N = \{1, \ldots, n\}$ be the set of claimants, $E \geq 0$ the endowment to be divided and $d \in \mathbb{R}_+^N$ the vector of claims such that $\sum_{i \in N} d_i \geq E$.

The truncated claim of a claimant $i \in N$ is the minimum of the claim and the endowment, that is, $t_i(E, d) = t_i = \min\{d_i, E\},\ i = 1, \ldots, n$.

Suppose that each agent claims specific parts of E equal to her/his claim. After arranging which parts agents claim so as to "minimize conflict", equal division prevails among all agents claiming a specific part and the minimal overlap rule (MO) assigns the sum of the compensations she/he gets from the various parts that he claimed.

Let $d_0 = 0$. For each problem (E, d) and each claimant $i \in N$,

1) If $E \leq d_n$ then

$$\mathrm{MO}_i(E, d) = \frac{t_1}{n} + \frac{t_2 - t_1}{n-1} + \cdots + \frac{t_i - t_{i-1}}{n-i+1}.$$

2) If $E > d_n$, let $s' \in (d_{k'}, d_{k'+1}]$, with $k' \in \{0, 1, \ldots, n-2\}$, be the unique solution to the equation $\sum\limits_{j \in N} \max\{d_j - s, 0\} = E - s$. Then,

$$\mathrm{MO}_i(E, d) = \begin{cases} \frac{d_1}{n} + \frac{d_2 - d_1}{n-1} + \cdots + \frac{d_i - d_{i-1}}{n-i+1} & \text{if } i \in \{1, \ldots, k'\} \\ \mathrm{MO}_i(s', d) + d_i - s' & \text{if } i \in \{k'+1, \ldots, n\} \end{cases}.$$

Value

The awards vector selected by the MO rule. If `name = TRUE`, the name of the function (MO) as a character string.

References

Mirás Calvo, M.Á., Núñez Lugilde, I., Quinteiro Sandomingo, C., and Sánchez-Rodríguez, E. (2023). Refining the Lorenz-ranking of rules for claims problems on restricted domains. International Journal of Economic Theory 19(3), 526-558.

O'Neill, B. (1982). A problem of rights arbitration from the Talmud. Mathemathical Social Sciences. 2, 345-371.

Thomson, W. (2019). How to divide when there isn't enough. From Aristotle, the Talmud, and Maimonides to the axiomatics of resource allocation. Cambridge University Press.

See Also

allrules, CD.

Examples

```
E=10
d=c(2,4,7,8)
MO(E,d)
```

`pathawards` *The path of awards for two claimants*

Description

This function returns the graphical representation of the path of awards of any rule for a claims vector and a pair of claimants.

Usage

```
pathawards(d, claimants, Rule, col = "red", points = 201)
```

Arguments

`d`	The vector of claims.
`claimants`	Two claimants.
`Rule`	The rule: AA, APRO, CE, CEA, CEL, AV, DT, MO, PIN, PRO, RA, Talmud, RTalmud.
`col`	The colour.
`points`	The number of values of the endowment to draw the path.

Details

Let $N = \{1, \ldots, n\}$ be the set of claimants, $d \in \mathbb{R}^N_+$ a vector of claims and denote by $D = \sum_{i \in N} d_i$ the sum of claims.

The path of awards of a rule $\mathcal{R}$ for two claimants $i, j \in N$ is the parametric curve:

$$p(E) = \Big\{(\mathcal{R}_i(E,d), \mathcal{R}_j(E,d)) \in \mathbb{R}^2 : \ E \in [0, D]\Big\}.$$

Value

The graphical representation of the path of awards of a rule for the given claims and a pair of claimants.

References

Thomson, W. (2019). How to divide when there isn't enough. From Aristotle, the Talmud, and Maimonides to the axiomatics of resource allocation. Cambridge University Press.

See Also

pathawards3, schedrule, schedrules, verticalruleplot.

Examples

```
d=c(2,4,7,8)
claimants=c(1,2)
Rule=Talmud
pathawards(d,claimants,Rule)
# The path of awards of the concede-and-divide rule
pathawards(c(2,3),c(1,2),CD)
#The path of awards of the DT rule for d=(d1,d2) with d2<2d1
pathawards(c(1,1.5),c(1,2),DT,col="blue",points=1001)
#The path of awards of the DT rule for d=(d1,d2) with d2>2d1
pathawards(c(1,2.5),c(1,2),DT,col="blue",points=1001)
```

---

pathawards3 *The path of awards for three claimants*

Description

This function returns the graphical representation of the path of awards of any rule for a claims vector and three claimants.

Usage

```
pathawards3(d, claimants, Rule, col = "red", points = 300)
```

Arguments

d	The vector of claims.
claimants	Three claimants.
Rule	The rule: AA, APRO, CE, CEA, CEL, AV, DT, MO, PIN, PRO, RA, Talmud, RTalmud.
col	The colour of the path, by default, col="red".
points	The number of values of the endowment to draw the path.

Details

Let $N = \{1, \dots, n\}$ be the set of claimants, $d \in \mathbb{R}^N_+$ a vector of claims and denote by $D = \sum_{i \in N} d_i$ the sum of claims.

The path of awards of a rule $\mathcal{R}$ for three claimants $i, j, k \in N$ is the parametric curve:

$$p(E) = \Big\{(\mathcal{R}_i(E,d), \mathcal{R}_j(E,d), \mathcal{R}_k(E,d)) \in \mathbb{R}^3 :\ E \in [0, D]\Big\}.$$

Value

The graphical representation of the path of awards of a rule for the given claims and three claimants.

See Also

pathawards, schedrule, schedrules, verticalruleplot.

Examples

```
d=c(2,4,7,8)
claimants=c(1,3,4)
Rule=Talmud
pathawards3(d,claimants,Rule)
```

`PIN` *Piniles' rule*

Description

This function returns the awards vector assigned by the Piniles' rule (PIN) to a claims problem.

Usage

```
PIN(E, d, name = FALSE)
```

Arguments

`E`	The endowment.
`d`	The vector of claims.
`name`	A logical value.

Details

Let $N = \{1, \ldots, n\}$ be the set of claimants, $E \geq 0$ the endowment to be divided and $d \in \mathbb{R}_+^N$ the vector of claims such that $D = \sum_{i \in N} d_i \geq E$.

The Piniles' rule (PIN) coincides with the constrained equal awards rule (CEA) applied to the problem $(E, d/2)$ if the endowment is less or equal than the half-sum of the claims, $D/2$. Otherwise it assigns to each claimant i half of the claim, $d_i/2$, and, then, it distributes the remainder with the CEA rule. Therefore, for each $i \in N$,

$$\text{PIN}_i(E, d) = \begin{cases} \min\{\frac{d_i}{2}, \lambda\} & \text{if } E \leq \frac{1}{2}D \\ \frac{d_i}{2} + \min\{\frac{d_i}{2}, \lambda\} & \text{if } E \geq \frac{1}{2}D \end{cases},$$

where $\lambda \geq 0$ is chosen such that $\sum_{i \in N} \text{PIN}_i(E, d) = E$.

Value

The awards vector selected by the PIN rule. If `name = TRUE`, the name of the function (PIN) as a character string.

References

Piniles, H.M. (1861). Darkah shel Torah. Forester, Vienna.

Thomson, W. (2019). How to divide when there isn't enough. From Aristotle, the Talmud, and Maimonides to the axiomatics of resource allocation. Cambridge University Press.

See Also

allrules, axioms, CEA, Talmud.

Examples

```
E=10
d=c(2,4,7,8)
PIN(E,d)
```

---

plotrule — *Plot of an awards vector*

Description

This function plots an awards vector in the set of awards vectors for a claims problem.

Usage

```
plotrule(E, d, Rule = NULL, awards = NULL, set = TRUE, col = "blue")
```

Arguments

E	The endowment.
d	The vector of claims.
Rule	A rule: AA, APRO, CE, CEA, CEL, AV, DT, MO, PIN, PRO, RA, Talmud, RTalmud.
awards	An awards vector.
set	A logical value.
col	The colour.

Details

Let $N = \{1, \ldots, n\}$ be the set of claimants, $E \geq 0$ the endowment to be divided and $d \in \mathbb{R}_+^N$ the vector of claims such that $\sum_{i \in N} d_i \geq E$.

A vector $x = (x_1, \ldots, x_n)$ is an awards vector for the claims problem (E, d) if $0 \leq x \leq d$ and satisfies the balance requirement, that is, $\sum_{i=1}^{n} x_i = E$. Let $X(E, d)$ be the set of awards vectors for the problem (E, d).

A rule is a function that assigns to each claims problem (E, d) an awards vector for (E, d), that is, a division between the claimants of the amount available.

Value

If `set = TRUE`, the function creates a new figure plotting both the set of awards vectors for the claims problem and the given awards vector. Otherwise, it just adds to the current picture the point representing the given awards vector. The function only plots one awards vector at a time.

The awards vector can be introduced directly as a vector. Alternatively, we can provide a rule and then the awards vector to be plotted is the one selected by the rule for the claims problem. Therefore, if `Rule = NULL` it plots the given awards vector. Otherwise, it plots the awards vector selected by the given rule for the claims problem. In order to plot two (or more) awards vectors, draw the first one with the option `set = TRUE` and add the others, one by one, with the option `set = FALSE`.

See Also

allrules, setofawards.

Examples

```
E=10
d=c(2,4,7,8)
plotrule(E,d,Rule=AA,col="red")
# Plotting the awards vector (1,3,5,1) and the AA rule
# First, plot the awards vector (1,3,5,1) and the set of awards
plotrule(E,d,awards=c(1,3,5,1),col="green")
# Second, add the AA rule with the option set=FALSE
plotrule(E,d,Rule=AA,set=FALSE,col="red")
```

`PRO` *Proportional rule*

Description

This function returns the awards vector assigned by the proportional rule (PRO) to a claims problem.

Usage

```
PRO(E, d, name = FALSE)
```

Arguments

`E`	The endowment.
`d`	The vector of claims.
`name`	A logical value.

Details

Let $N = \{1, \ldots, n\}$ be the set of claimants, $E \geq 0$ the endowment to be divided and $d \in \mathbb{R}_+^N$ the vector of claims such that $D = \sum_{i \in N} d_i \geq E$.

The proportional rule (PRO) distributes awards proportional to claims, that is,

$$\text{PRO}(E, d) = \frac{E}{D} d.$$

Value

The awards vector selected by the PRO rule. If `name = TRUE`, the name of the function (PRO) as a character string.

References

Aristotle, Ethics, Thompson, J.A.K., tr. 1985. Harmondsworth: Penguin.

Thomson, W. (2019). How to divide when there isn't enough. From Aristotle, the Talmud, and Maimonides to the axiomatics of resource allocation. Cambridge University Press.

See Also

allrules, APRO, axioms.

Examples

```
E=10
d=c(2,4,7,8)
PRO(E,d)
```

problemdata *Claims problem data*

Description

The function returns which of the following subdomains the claims problem belongs to: the lower-half, higher-half, and midpoint domains. In addittion, the function returns the minimal rights vector, the truncated claims vector, the sum and the half-sum of claims.

Usage

```
problemdata(E, d, draw = FALSE)
```

Arguments

E	The endowment.
d	The vector of claims.
draw	A logical value.

Details

Let $N = \{1, \ldots, n\}$ be the set of claimants, $E \geq 0$ the endowment to be divided and $d \in \mathbb{R}_+^N$ the vector of claims such that $D = \sum_{i \in N} d_i \geq E$.

The lower-half domain is the subdomain of claims problems for which the endowment is less or equal than the half-sum of claims, $E \leq D/2$.

The higher-half domain is the subdomain of claims problems for which the endowment is greater or equal than the half-sum of claims, $E \geq D/2$.

The midpoint domain is the subdomain of claims problems for which the endowment is equal to the half-sum of claims, $E = D/2$.

The minimal right of claimant $i \in N$ in (E, d) is whatever is left after every other claimant has received his claim, or 0 if that is not possible:

$$m_i(E, d) = \max\{0, E - d(N \backslash \{i\})\}.$$

Let $m(E, d) = \Big(m_1(E, d), \ldots, m_n(E, d)\Big)$ be the vector of minimal rights.

The truncated claim of claimant $i \in N$ in (E, d) is the minimum of the claim and the endowment:

$$t_i(E, d) = \min\{d_i, E\}.$$

Let $t(E, d) = \Big(t_1(E, d), \ldots, t_n(E, d)\Big)$ be the vector of truncated claims.

Value

The minimal rights vector; the truncated claims vector; the sum, the half-sum of the claims, and the class (lower-half, higher-half, and midpoint domains) to which the claims problem belongs. It returns `cod = 1` if the claims problem belong to the lower-half domain, `cod = -1` if it belongs to the higher-half domain, and `cod = 0` for the midpoint domain. Moreover, if `draw = TRUE` a plot of the claims, from small to large in the interval [0,D], is given.

See Also

allrules, setofawards.

Examples

```
E=10
d=c(2,4,7,8)
problemdata(E,d,draw=TRUE)
```

RA *Random arrival rule*

Description

This function returns the awards vector assigned by the random arrival rule (RA) to a claims problem.

Usage

```
RA(E, d, name = FALSE)
```

Arguments

E	The endowment.
d	The vector of claims.
name	A logical value.

Details

Let $N = \{1, \ldots, n\}$ be the set of claimants, $E \geq 0$ the endowment to be divided and $d \in \mathbb{R}_+^N$ the vector of claims such that $\sum_{i \in N} d_i \geq E$. For each coalition $S \in 2^N$, $d(S) = \sum_{j \in S} d_j$.

The random arrival rule (RA) considers all the possible arrivals of the claimants and applies the principle “first to arrive, first to be served". Then, for each order, the corresponding marginal worth vector assigns to each claimant the minimum of her/his claim and what remains of the endowment. The rule averages all the marginal worth vectors considering all the permutations of the elements of N.

Let Π^N denote the set of permutations of the set of claimants N and $|\Pi^N|$ its cardinality. Given a permutation $\pi \in \Pi$ and a claimant $i \in N$ let $\pi_{\leq i}$ be the set of claimants that precede i in the order π, that is, $\pi_{\leq i} = \{j \in N : \pi(j) < \pi(i)\}$.

The random arrival rule assigns to each (E, d) and each $i \in N$,

$$\text{RA}_i(E, d) = \frac{1}{|\Pi^N|} \sum_{\pi \in \Pi^N} \min\Big\{d_i, \max\{0, E - d(\pi_{\leq i})\}\Big\}.$$

The random arrival rule corresponds to the Shapley value of the associated (pessimistic) coalitional game.

This function is programmed following the algorithm of Le Creurer et al. (2022).

Value

The awards vector selected by the RA rule. If name = TRUE, the name of the function (RA) as a character string.

References

Le Creurer, I.J, Mirás Calvo, M. A., Núñez Lugilde, I., Quinteiro Sandomingo, C., and Sánchez Rodríguez, E. (2022). On the computation of the Shapley value and the random arrival rule. Available at https://papers.ssrn.com/sol3/papers.cfm?abstract_id=4293746.

O'Neill, B. (1982) A problem of rights arbitration from the Talmud. Mathematical Social Sciences 2, 345–371.

Thomson, W. (2019). How to divide when there isn't enough. From Aristotle, the Talmud, and Maimonides to the axiomatics of resource allocation. Cambridge University Press.

See Also

AA, allrules, APRO, axioms, CD, setofawards, Talmud.

Examples

```
E=10
d=c(2,4,7,8)
RA(E,d)
D=sum(d)
#The random arrival rule is self-dual: RA(E,d)= d-RA(D-E,d)
d-RA(D-E,d)
```

RTalmud *Reverse Talmud rule*

Description

This function returns the awards vector assigned by the reverse Talmud rule to a claims problem.

Usage

```
RTalmud(E, d, name = FALSE)
```

Arguments

E	The endowment.
d	The vector of claims.
name	A logical value.

Details

Let $N = \{1, \ldots, n\}$ be the set of claimants, $E \geq 0$ the endowment to be divided and $d \in \mathbb{R}_+^N$ the vector of claims such that $D = \sum_{i \in N} d_i \geq E$.

The reverse Talmud rule (RTalmud) coincides with the constrained equal losses rule (CEL) applied to the problem $(E, d/2)$ if the endowment is less or equal than the half-sum of the claims, $D/2$. Otherwise, the reverse Talmud rule assigns $d/2$ and the remainder, $E - D/2$, is awarded with the constrained equal awards rule with claims $d/2$. Therefore, for each $i \in N$,

$$\text{RTalmud}_i(E, d) = \begin{cases} \max\{\frac{d_i}{2} - \lambda, 0\} & \text{if } E \leq \frac{1}{2}D \\ \frac{d_i}{2} + \min\{\frac{d_i}{2}, \lambda\} & \text{if } E \geq \frac{1}{2}D \end{cases},$$

where $\lambda \geq 0$ is chosen such that $\sum\limits_{i \in N} \text{RTalmud}_i(E, d) = E$.

Value

The awards vector selected by the reverse Talmud rule. If `name = TRUE`, the name of the function (RTalmud) as a character string.

References

Chun, Y., Schummer, J., and Thomson, W. (2001). Constrained egalitarianism: a new solution for claims problems. Seoul Journal of Economics 14, 269-297.

Thomson, W. (2019). How to divide when there isn't enough. From Aristotle, the Talmud, and Maimonides to the axiomatics of resource allocation. Cambridge University Press.

See Also

AA, allrules, APRO, Talmud, CEA, CEL, CD, RA.

Examples

```
E=10
d=c(2,4,7,8)
RTalmud(E,d)
```

`schedrule` *Schedules of awards of a rule*

Description

This function returns the graphical representation of the schedules of awards of any rule for a claims vector.

Usage

```
schedrule(d, claimants, Rule, col = NULL, points = 201, legend = TRUE)
```

Arguments

`d`	A vector of claims.
`claimants`	A subset of claimants.
`Rule`	The rule: AA, APRO, CE, CEA, CEL, AV, DT, MO, PIN, PRO, RA, Talmud, RTalmud.
`col`	The colours. If `col = NULL` then the sequence of default colours is chosen randomly.
`points`	The number of values of the endowment to draw the path.
`legend`	A logical value. The colour legend is shown if `legend = TRUE`.

Details

Let $N = \{1, \ldots, n\}$ be the set of claimants, $d \in \mathbb{R}^N_+$ a vector of claims and denote by $D = \sum_{i \in N} d_i$ the sum of the claims.

The schedules of awards of a rule $\mathcal{R}$ for claimant i is the function S that assigns to each $E \in [0, D]$ the value: $S(E) = \mathcal{R}_i(E, d) \in \mathbb{R}$. Therefore, the schedules of awards of a rule plots each claimants's award as a function of E.

Value

The graphical representation of the schedules of awards of a rule for a claims vector and a group of claimants.

References

Thomson, W. (2019). How to divide when there isn't enough. From Aristotle, the Talmud, and Maimonides to the axiomatics of resource allocation. Cambridge University Press.

See Also

pathawards, pathawards3, schedrules, verticalruleplot.

Examples

```
d=c(2,4,7,8)
Rule=Talmud
claimants=c(1,2,3,4)
col=c("red","green","yellow","blue")
schedrule(d,claimants,Rule,col)
# The schedules of awards of the concede-and-divide rule.
schedrule(c(2,4),c(1,2),CD)
```

`schedrules` *Schedules of awards of several rules*

Description

This function returns the graphical representation of the schedules of awards of different rules for a claims vector and a given claimant.

Usage

```
schedrules(d, claimant, Rules, col = NULL, points = 201, legend = TRUE)
```

Arguments

`d`	A vector of claims.
`claimant`	A claimant.
`Rules`	The rules: AA, APRO, CE, CEA, CEL, AV, DT, MO, PIN, PRO, RA, Talmud, RTalmud.
`col`	The colours. If `col = NULL` then the sequence of default colours is: c("red", "blue", "green", "yellow", "pink", "orange", "coral4", "darkgray", "burlywood3", "black", "darkorange", "darkviolet").
`points`	The number of endowment values to draw the path.
`legend`	A logical value. The colour legend is shown if `legend = TRUE`.

Details

Let $N = \{1, \ldots, n\}$ be the set of claimants, $d \in \mathbb{R}_+^N$ a vector of claims and denote by $D = \sum_{i \in N} d_i$ the sum of the claims.

The schedules of awards of a rule $\mathcal{R}$ for claimant i is the function S that assigns to each $E \in [0, D]$ the value: $S(E) = \mathcal{R}_i(E, d) \in \mathbb{R}$. Therefore, the schedules of awards of a rule plots each claimants's award as a function of E.

Value

The graphical representation of the schedules of awards of the rules for the claims vector and the same claimant.

References

Thomson, W. (2019). How to divide when there isn't enough. From Aristotle, the Talmud, and Maimonides to the axiomatics of resource allocation. Cambridge University Press.

See Also

pathawards, pathawards3, schedrule, verticalruleplot.

Examples

```
d=c(2,4,7,8)
claimant=2
Rules=c(Talmud,RA,AA)
col=c("red","green","blue")
schedrules(d,claimant,Rules,col)
```

---

setofawards *Set of awards vectors for a claims problem*

Description

This function plots the set of awards vectors for a claims problem with 2, 3, or 4 claimants and returns its vertices for any problem.

Usage

```
setofawards(E, d, draw = TRUE, col = NULL)
```

Arguments

`E`	The endowment.
`d`	The vector of claims.
`draw`	A logical value.
`col`	The colour.

Details

Let $N = \{1, \ldots, n\}$ be the set of claimants, $E \geq 0$ the endowment to be divided and $d \in \mathbb{R}^N_+$ the vector of claims such that $\sum_{i \in N} d_i \geq E$.

A vector $x = (x_1, \ldots, x_n)$ is an awards vector for the claims problem (E, d) if $0 \leq x \leq d$ and satisfies the balance requirement, that is, $\sum_{i=1}^n x_i = E$. Let $X(E, d)$ be the set of awards vectors for the problem (E, d).

For each subset S of the set of claimants N, let $d(S) = \sum_{j \in S} d_j$ be the sum of claims of the members of S and let $N \backslash S$ be the complementary coalition of S.

The minimal right of claimant $i \in N$ in (E, d) is whatever is left after every other claimant has received his claim, or 0 if that is not possible:

$$m_i(E, d) = \max\{0, E - d(N \backslash \{i\})\},\ i = 1, \ldots, n.$$

Let $m(E, d) = \Big(m_1(E, d), \ldots, m_n(E, d)\Big)$ be the vector of minimal rights.

The truncated claim of claimant $i \in N$ in (E, d) is the minimum of the claim and the endowment:

$$t_i(E, d) = \min\{d_i, E\},\ i = 1, \ldots, n.$$

Let $t(E,d) = \Big(t_1(E,d),\ldots,t_n(E,d)\Big)$ be the vector of truncated claims.

A vector x is efficient if the sum of its coordinates coincides with the endowment. The set of awards is the the set of all efficient vectors bounded by the minimal right and trucated claim vectors.

The set of awards vectors for the claims problem (E,d) can be given in terms of the minimal rights and truncated claims vectors:

$$X(E,d) = \Big\{x \in \mathbb{R}^n : x(N) = E, m(E,d) \leq x \leq t(E,d)\Big\}.$$

The set of awards vectors for a problem coincides with the core of its associated coalitional (pessimistic) game.

The vertices of the set of awards are the marginal worth vectors. For each order of the claimants, the marginal worth vectors are obtained applying the principle “first to arrive, first to be served". Then, for each order, the corresponding marginal worth vector assigns to each claimant the minimum of her/his claim and what remains of the endowment.

Value

The vertices of the set of awards vectors for any claims problem. For two-claimant and three-claimant problems, if `draw = TRUE` it plots the set of awards vectors. For a four-claimant problem, if `draw = TRUE`, it plots the projection of the set of awards vector over the euclidean space of the first three coordinates. For a claims problem with more than four claimants, it only displays the vertices of the set of awards. The default colours (`col = NULL`) are: red for two-claimant problems, beige for three-claimant problems, and white for four-claimant problems.

References

Mirás Calvo, M.A., Núñez Lugilde, I., Quinteiro Sandomingo, C., and Sánchez-Rodríguez, E. (2024). On properties of the set of awards vectors for a claims problem. TOP 32, 137-167.

Thomson, W. (2019). How to divide when there isn’t enough. From Aristotle, the Talmud, and Maimonides to the axiomatics of resource allocation. Cambridge University Press.

See Also

AA, plotrule, problemdata, RA, volume.

Examples

```
E=10
d=c(2,4,7,8)
setofawards(E,d,col="darkgreen")
```

`Talmud` *Talmud rule*

Description

This function returns the awards vector assigned by the Talmud rule to a claims problem.

Usage

```
Talmud(E, d, name = FALSE)
```

Arguments

`E`	The endowment.
`d`	The vector of claims.
`name`	A logical value.

Details

Let $N = \{1, \ldots, n\}$ be the set of claimants, $E \geq 0$ the endowment to be divided and $d \in \mathbb{R}_+^N$ the vector of claims such that $D = \sum_{i \in N} d_i \geq E$.

The Talmud rule (Talmud) coincides with the constrained equal awards rule (CEA) applied to the problem $(E, d/2)$ if the endowment is less or equal than the half-sum of the claims, $D/2$. Otherwise, the Talmud rule assigns $d/2$ and the remainder, $E - D/2$, is awarded with the constrained equal losses rule with claims $d/2$. Therefore, for each $i \in N$,

$$\text{Talmud}_i(E, d) = \begin{cases} \min\{\frac{d_i}{2}, \lambda\} & \text{if } E \leq \frac{1}{2}D \\ d_i - \min\{\frac{d_i}{2}, \lambda\} & \text{if } E \geq \frac{1}{2}D \end{cases},$$

where $\lambda \geq 0$ is chosen such that $\sum\limits_{i \in N} \text{Talmud}_i(E, d) = E$.

The Talmud rule when applied to a two-claimant problem is often referred to as the contested garment rule and coincides with concede-and-divide rule. The Talmud rule corresponds to the nucleolus of the associated (pessimistic) coalitional game.

Value

The awards vector selected by the Talmud rule. If `name = TRUE`, the name of the function (Talmud) as a character string.

References

Aumann, R. and Maschler, M. (1985). Game theoretic analysis of a bankruptcy problem from the Talmud. Journal of Economic Theory 36, 195-213.

Thomson, W. (2019). How to divide when there isn't enough. From Aristotle, the Talmud, and Maimonides to the axiomatics of resource allocation. Cambridge University Press.

See Also

AA, allrules, APRO, axioms, CEA, CEL, CD, RA, RTalmud.

Examples

```
E=10
d=c(2,4,7,8)
Talmud(E,d)
D=sum(d)
#The Talmud rule is self-dual
d-Talmud(D-E,d)
```

`Universityfunds` *University funds data*

Description

Data to distribute a certain amount of money to buy equipment for teaching laboratories among the different degree courses in the Universidad Miguel Hernández of Elche (Spain)

Usage

```
Universityfunds
```

Format

A data frame with 27 rows and 3 variables:

[,1]	`Degree`	categorical	Degree courses in a University
[,2]	`Entitlement`	numeric	Objective entitlements based on number of students, experimental level, ... (euros)
[,3]	`Claim`	numeric	Claim of each degree (euros)

Source

The data were obtained from Pulido et al (2002).

References

Pulido, M., Sánchez Soriano, J., and Llorca, N. (2002). Game Theory Techniques for University Management: An Extended Bankruptcy Model. Annals of Operations Research, 109, 129-142.

Pulido, M., Borm, P, Hendricks, R, Llorca, N., and Sánchez Soriano, J. (2008). Compromise solutions for bankruptcy situations with references. Annals of Operations Research, 158, 133-141.

Núñez Lugilde, I., Estévez Fernández, A., and Sánchez Rodríguez, E. (2024). Priority coalitional games and claims problems. Mathematical Methods of Operations Research, 100, 669–701.

Examples

```
data(Universityfunds)
head(Universityfunds)

E = 717293.11
d <- Universityfunds$Claim-Universityfunds$Entitlement
E = E-sum(Universityfunds$Entitlement)
Universityfunds$Entitlement+CEA(E,d)
Universityfunds$Entitlement+PRO(E,d)
```

---

verticalruleplot *Vertical rule plot*

Description

For each claimant, it plots a vertical line with his claim and a point on the awards vector of the chosen rules.

Usage

```
verticalruleplot(E, d, Rules, col = NULL, legend = TRUE)
```

Arguments

`E`	The endowment.
`d`	The vector of claims
`Rules`	The rules: AA, APRO, CE, CEA, CEL, AV, DT, MO, PIN, PRO, RA, Talmud, or RTalmud.
`col`	The colours. If `col=NULL` then the sequence of default colours is: c("red", "blue", "green", "yellow", "pink", "orange", "coral4", "darkgray", "burlywood3", "black", "darkorange", "darkviolet").
`legend`	A logical value. The colour legend is shown if `legend=TRUE`.

Details

Let $N = \{1, \ldots, n\}$ be the set of claimants, $E \geq 0$ the endowment to be divided and $d \in \mathbb{R}_+^N$ the vector of claims such that $\sum_{i \in N} d_i \geq E$.

A vector $x = (x_1, \ldots, x_n)$ is an awards vector for the claims problem (E, d) if $0 \leq x \leq d$ and satisfies the balance requirement, that is, $\sum_{i=1}^{n} x_i = E$.

A rule is a function that assigns to each claims problem (E, d) an awards vector, that is, a division between the claimants of the amount available.

The formal definitions of the main rules are given in the corresponding function help.

Value

This function represents the claims vector and the awards vector assigned by several rules as vertical segments.

References

Thomson, W. (2019). How to divide when there isn't enough. From Aristotle, the Talmud, and Maimonides to the axiomatics of resource allocation. Cambridge University Press.

See Also

allrules, pathawards, pathawards3, schedrule, schedrules.

Examples

```
E=10
d=c(2,4,7,8)
Rules=c(Talmud,RA,AA)
col=c("red","green","blue")
verticalruleplot(E,d,Rules,col)
```

`volume` *Volume of the set of awards vectors*

Description

This function computes the volume of the set of award vectors of a claims problem and the projected volume.

Usage

```
volume(E, d, real = TRUE)
```

Arguments

`E`	The endowment.
`d`	The vector of claims.
`real`	Logical parameter. By default, `real = TRUE`.

Details

Let $N = \{1, \ldots, n\}$ be the set of claimants, $E \geq 0$ the endowment to be divided and $d \in \mathbb{R}^N_+$ the vector of claims such that $\sum_{i \in N} d_i \geq E$.

A vector $x = (x_1, \ldots, x_n)$ is an awards vector for the claims problem (E, d) if $0 \leq x \leq d$ and satisfies the balance requirement, that is, $\sum_{i=1}^{n} x_i = E$. Let $X(E, d)$ be the set of awards vectors for (E, d).

Let μ be the $(n-1)$-dimensional Lebesgue measure. We define by $V(E, d) = \mu(X(E, d))$ the measure (volume) of the set of awards $X(E, d)$ and $\hat{V}(E, d)$ the volume of the projection onto an $(n-1)$-dimensional space.

$$V(E, d) = \sqrt{n}\hat{V}(E, d).$$

The function is programmed following the procedure explained in Mirás Calvo et al. (2024b).

Value

The volume of the set of awards vectors. If `real = FALSE`, it returns the volume of the projection into the last coordinate.

References

Mirás Calvo, M.A., Núñez Lugilde, I., Quinteiro Sandomingo, C., and Sánchez-Rodríguez, E. (2024a). An algorithm to compute the average-of-awards rule for claims problems with an application to the allocation of CO_2 emissions. Annals of Operations Research, 336: 1435-1459.

Mirás Calvo, M.A., Núñez Lugilde, I., Quinteiro Sandomingo, C., and Sánchez-Rodríguez, E. (2024b). On properties of the set of awards vectors for a claims problem. TOP, 32: 137-167.

See Also

setofawards.

Examples

```
E=10
d=c(2,4,7,10)
volume(E,d)
#The volume function is a symmetric function.
D=sum(d)
volume(D-E,d)
```

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