## Package 'CircMLE'

## July 21, 2025

Title Maximum Likelihood Analysis of Circular Data

Version 0.3.0

Description A series of wrapper functions to

implement the 10 maximum likelihood models of animal orientation
described by Schnute and Groot (1992) <DOI:10.1016/S0003-3472(05)80068-5>. The
functions also include the ability to use different optimizer
methods and calculate various model selection metrics (i.e., AIC,
AICc, BIC). The ability to perform variants of the Hermans-Rasson test and
Pycke test is also included as described in Landler et al. (2019)
<DOI:10.1186/s12898-019-0246-8>. The latest version also includes a new
method to calculate circular-circular and circular-linear distance correlations.

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check\_data

Data Checking Function

## Description

Make sure data is in the right format. Datasets are coerced into class 'circular' of type = angles, units = radians, and modulo = 2pi. It is recommended to set these atributes ahead of time.

## Usage

check\_data(data)

## Arguments

data A vector, class 'circular' is recommended but not required

## Examples

```
testdata = circular::rvonmises(100, mu = circular::circular(pi), kappa = 3,
    control.circular = list(units = "degrees"))
check_data(testdata)
```

circ\_mle

## circ\_mle

## Description

Run all 10 maximum likelihood models of circular orientation

## Usage

```
circ_mle(
  data,
  criterion = "AIC",
  nchains = 5,
  BadStart = 10^9,
  niter = 5000,
  method = "BFGS",
  lambda.min = 0.25,
  q.diff,
  exclude = NULL
)
```

## Arguments

data	A vector of class 'circular'
criterion	chose from either "AIC", "AICc", or "BIC" for the model comparison informa- tion criterion. (default = "AIC")
nchains	A positive integer indicating the number of chains to run. Only the chain with the lowest log likelihood is returned (default = $5$ )
BadStart	An integer to replace the log likelihood when starting parameters of the optimizer fall outside the preset bounds. This is usually set to a large number. Can also be set to Inf, but will result in an error if a method other than "Nelder-Mead" is chosen. (default = $10^{9}$ )
niter	The maximum number of iterations for the optimizing algorithm. Equivalent to the 'maxit' control parameter of the optim() function. See ?optim for more details. (default = $5000$ )
method	A character string indicating the optimizing algorithm to use. Either "BFGS" or "Nelder-Mead" are recommended. See ?optim for more details. (default = "BFGS")
lambda.min	The minimum proportional size of the first distribution. Must be between 0 and 1. (default = $0.25$ )
q.diff	The minimum difference (in radians) in preferred direction for bimodal models. Must be set between 0 and pi. (default = $pi/4$ )
exclude	A character vector of the models to be excluded from the calculations. (The default is to include all 10 models). For example, exclude = $c("M1", "M3A", "M5B")$ .

#### Value

A list with 4 elements:

\$results: A data frame consisting of a row for each model (rownames) with the columns: 1 = number of free parameters, 2 = mu1, 3 = kappa1, 4 = lamda, 5 = mu2, 6 = kappa2, 7 = negative log likelihood, <math>8 = Counts.function, 9 = Counts.Gradient, 10 = convergence, 11 = message, 12 = AIC, 13 = AICc, 14 = BIC, 15 = delta AIC, 16 = delta AICc, 17 = delta BIC, 18 = relative likelihoods of criterion chosen, 19 = model weights (probabilities) for criterion chosen, 20 = evidence ratios for the best model selected by the criterion. See ?optim for details on columns 8-11.

\$bestmodel: The best model according to the criterion chosen

\$rt: A two-element vector giving the test statistic and p-value for the Rayleigh Test

\$hessians: A list with each element containing the hessian matrix for each model. Used for calculating connfidence intervals of parameters.

#### Examples

```
testdata = circular::rvonmises(100, mu = circular::circular(pi), kappa = 3)
circ_mle(testdata)
```

ci\_circmle

ci\_circmle

#### Description

Calculate the 95% confidence interval for estimated model parameters

#### Usage

ci\_circmle(circmle, model)

#### Arguments

circmle	A list consisting of the output from function 'circ_mle'
model	character string indicating the model to be used to estimate parameter uncer-
	tainty. Must be one of c("M2A", "M2B", "M2C", "M3A", "M3B", "M4A",
	"M4B", "M5A", "M5B"). Default = the \$bestmodel from the circmle object.

#### Value

A data frame with a column for the parameter name, the maximum likelihood estimate (MLE), standard error (SE), and 95% confidence interval (CI) for each estimated model parameter.

#### Examples

```
testdata = circular::rvonmises(100, mu = circular::circular(pi), kappa = 3)
fit <- circ_mle(testdata)
ci_circmle(fit)</pre>
```

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dcor.circular

## Description

Perform a distance correlation between circular datasets or between circular and linear datasets.

#### Usage

```
dcor.circular(x, y, method = "chord", type = "c-c", ...)
```

## Arguments

x	A vector of class 'circular', or numeric vector of angles measured in radians
У	A vector of class 'circular', numeric vector of angles measured in radians, or numeric vector
method	the distance measure to be used. This must be one of the following func- tions: '"angularseparation"', '"chord"', '"geodesic"', or '"circ.range"' (default = "chord"). see ?dist.circular for additional details.
type	if 'type == "c-c"' then perform a circular-circular distance corellation, else if 'type == "c-l"' then perform a circular-linear distance corellation (default = "c-c").
	additional parameters passed to the dcor.test function

#### Value

Same as from the dcor.test function: a list with class 'htest' containing

method: description of test

statistic: observed value of the test statistic

estimate: dCov(x,y) or dCor(x,y)

estimates: a vector: [dCov(x,y), dCor(x,y), dVar(x), dVar(y)]

replicates: replicates of the test statistic

p.value: approximate p-value of the test

n: sample size

data.name: description of data

## See Also

dcor dcov DCOR dcor.test dist.circular

#### Examples

```
# Circular-circular distance corellation
x <- circular::rvonmises(n = 50, mu = circular::circular(0), kappa = 3)
y <- x + circular::rvonmises(n = 50, mu = circular::circular(pi), kappa = 10)
dcor.circular(x, y)
# Run permutation test with 9999 iterations
dcor.circular(x, y, R = 9999)
# Circular-linear distance corellation
x <- circular::rvonmises(n = 50, mu = circular::circular(0), kappa = 3)
y <- as.numeric(x) + rnorm(50, mean = 5, sd = 2)
dcor.circular(x, y, type = "c-1", R = 9999)
```

HR\_test

#### Hermans-Rasson test function

#### Description

Perform variants of the Hermans-Rasson test.

#### Usage

HR\_test(data, original = F, iter = 9999)

## Arguments

data	A vector of class 'circular', or numeric vector of angles measured in radians
original	A logical of whether or not to run the original version of the Hermans-Rasson test or the newer version described in Landler et al. (2019) doi: $10.1186/s12898-019-0246-8$ (default = F)
iter	The number of bootstrap replicates to perform in order to estimate the p-value of the test. (default = 9999)

## Value

A numeric vector of the test statistic (T) and associated p-value

## Examples

```
testdata = circular::rvonmises(20, mu = circular::circular(pi), kappa = 3)
HR_test(testdata, iter = 999)
```

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lr\_test

## Description

Statistically test nested models for the rejection of the null model in favor of the alternative model.

#### Usage

lr\_test(data, null\_model, alt\_model)

## Arguments

data	A vector of class 'circular'
null_model	character string indicating the null model to be used. Must be one of c("M1", "M2A", "M2B", "M2C", "M3A", "M3B", "M4A", "M4B", "M5A", "M5B").
alt_model	character string indicating the alternative model to be used. Must be one of c("M1", "M2A", "M2B", "M2C", "M3A", "M3B", "M4A", "M4B", "M5A", "M5B").

## Value

A list with elements:

\$null\_model: Vector with the model name and number of free parameters

\$alt\_model: Vector with the model name and number of free parameters

\$lr: likelihood ratio test statistic 2 \* (-log(L1) - -log(L2))

\$df: degrees of freedom (difference in number of parameters between models)

\$p\_val: probability of rejecting null model in favor of alternative model due to chance (e.g, if <0.05, data favors alternative model).

## Examples

```
testdata = circular::rvonmises(100, mu = circular::circular(pi), kappa = 3)
lr_test(testdata, "M1", "M2A")
```

## Description

Run Maximum likelihood estimation for model M1.

## Usage

M1(data)

## Arguments

data

A vector of class 'circular'

## Value

A list with the elements:

\$lik: The negative log likelihood of data for model M1

## Examples

```
testdata = circular::rvonmises(100, mu = circular::circular(pi), kappa = 3)
M1(testdata)
```

M2A

## Model M2A MLE function

## Description

Run Maximum likelihood estimation for model M2A.

## Usage

```
M2A(data, BadStart, nchains, method, niter)
```

## Arguments

data	A vector of class 'circular'
BadStart	An integer to replace the log likelihood when starting parameters of the optimizer fall outside the preset bounds. This is usually set to a large number. Can also be set to Inf, but will result in an error if a method other than "Nelder-Mead" is chosen. (default = $10^{9}$ )
nchains	A positive integer indicating the number of chains to run. Only the chain with the lowest log likelihood is returned (default = $5$ )

M1

method	A character string indicating the optimizing algorithm to use. Either "BFGS" or "Nelder-Mead" are recommended. See ?optim for more details. (default = "BFGS")
niter	The maximum number of iterations for the optimizing algorithm. Equivalent to the 'maxit' control parameter of the optim() function. See ?optim for more details. (default = 5000)

### Value

A list with elements (same as for function optim()):

\$par: Vector with the optimized mean angle (mu1) and concentration parameter (kappa1)

\$lik: The negative log likelihood

\$counts: A two-element integer vector giving the number of calls to 'fn' and 'gr' respectively. See ?optim() for details.

\$convergence: An integer code. '0' indicates successful completion (which is always the case for "SANN"' and "Brent"'). Possible error codes are:

'1' indicates that the iteration limit 'maxit' had been reached.

'10' indicates degeneracy of the Nelder-Mead simplex.

'51' indicates a warning from the "L-BFGS-B"' method; see component 'message' for further details.

'52' indicates an error from the "L-BFGS-B"' method; see component 'message' for further details.

\$message: A character string giving any additional information returned by the optimizer, or 'NULL'.

#### Examples

```
testdata = circular::rvonmises(100, mu = circular::circular(pi), kappa = 3)
M2A(testdata)
```

M2B

Model M2B MLE function

#### Description

Run Maximum likelihood estimation for model M2B.

#### Usage

M2B(data, BadStart, nchains, method, niter)

#### Arguments

data	A vector of class 'circular'
BadStart	An integer to replace the log likelihood when starting parameters of the opti- mizer fall outside the preset bounds. This is usually set to a large number. Can also be set to Inf, but will result in an error if a method other than "Nelder-Mead" is chosen. (default = $10^{9}$ )
nchains	A positive integer indicating the number of chains to run. Only the chain with the lowest log likelihood is returned (default = $5$ )
method	A character string indicating the optimizing algorithm to use. Either "BFGS" or "Nelder-Mead" are recommended. See ?optim for more details. (default = "BFGS")
niter	The maximum number of iterations for the optimizing algorithm. Equivalent to the 'maxit' control parameter of the optim() function. See ?optim for more details. (default = 5000)

## Value

A list with elements (same as for function optim()):

\$par: Vector with the optimized mean angle (mu1) and concentration parameter (kappa1)

\$lik: The negative log likelihood

\$counts: A two-element integer vector giving the number of calls to 'fn' and 'gr' respectively. See ?optim() for details.

\$convergence: An integer code. '0' indicates successful completion (which is always the case for "SANN" and "Brent"). Possible error codes are:

'1' indicates that the iteration limit 'maxit' had been reached.

'10' indicates degeneracy of the Nelder-Mead simplex.

'51' indicates a warning from the "L-BFGS-B"' method; see component 'message' for further details.

'52' indicates an error from the "L-BFGS-B"' method; see component 'message' for further details.

\$message: A character string giving any additional information returned by the optimizer, or 'NULL'.

## Examples

```
testdata = circular::rvonmises(100, mu = circular::circular(pi), kappa = 3)
M2B(testdata)
```

#### Description

Run Maximum likelihood estimation for model M2C.

#### Usage

M2C(data, BadStart, nchains, method, niter, lambda.min)

#### Arguments

data	A vector of class 'circular'
BadStart	An integer to replace the log likelihood when starting parameters of the opti- mizer fall outside the preset bounds. This is usually set to a large number. Can also be set to Inf, but will result in an error if a method other than "Nelder-Mead" is chosen. (default = $10^{9}$ )
nchains	A positive integer indicating the number of chains to run. Only the chain with the lowest log likelihood is returned (default = $5$ )
method	A character string indicating the optimizing algorithm to use. Either "BFGS" or "Nelder-Mead" are recommended. See ?optim for more details. (default = "BFGS")
niter	The maximum number of iterations for the optimizing algorithm. Equivalent to the 'maxit' control parameter of the optim() function. See ?optim for more details. (default = 5000)
lambda.min	The minimum proportional size of the first distribution. Must be between 0 and 1. (default = $0.25$ )

#### Value

A list with elements (same as for function optim()):

\$par: Vector with the optimized mean angle (mu1), concentration parameter (kappa1) and proportional size (lambda) of the first distribution (mu1 and kappa1).

\$lik: The negative log likelihood

\$counts: A two-element integer vector giving the number of calls to 'fn' and 'gr' respectively. See ?optim() for details.

\$convergence: An integer code. '0' indicates successful completion (which is always the case for '"SANN"' and '"Brent"'). Possible error codes are:

'1' indicates that the iteration limit 'maxit' had been reached.

'10' indicates degeneracy of the Nelder-Mead simplex.

'51' indicates a warning from the "L-BFGS-B"' method; see component 'message' for further details.

'52' indicates an error from the "L-BFGS-B"' method; see component 'message' for further details.

\$message: A character string giving any additional information returned by the optimizer, or 'NULL'.

#### M2C

#### Examples

```
testdata = circular::rvonmises(100, mu = circular::circular(pi), kappa = 3)
M2C(testdata)
```

M3A

#### Model M3A MLE function

#### Description

Run Maximum likelihood estimation for model M3A.

#### Usage

M3A(data, BadStart, nchains, method, niter)

#### Arguments

data	A vector of class 'circular'
BadStart	An integer to replace the log likelihood when starting parameters of the opti- mizer fall outside the preset bounds. This is usually set to a large number. Can also be set to Inf, but will result in an error if a method other than "Nelder-Mead" is chosen. (default = $10^{9}$ )
nchains	A positive integer indicating the number of chains to run. Only the chain with the lowest log likelihood is returned (default = $5$ )
method	A character string indicating the optimizing algorithm to use. Either "BFGS" or "Nelder-Mead" are recommended. See ?optim for more details. (default = "BFGS")
niter	The maximum number of iterations for the optimizing algorithm. Equivalent to the 'maxit' control parameter of the optim() function. See ?optim for more details. (default = 5000)

## Value

A list with elements (same as for function optim()):

\$par: Vector with the optimized mean angle (mu1) and concentration parameter (kappa1).

\$lik: The negative log likelihood

\$counts: A two-element integer vector giving the number of calls to 'fn' and 'gr' respectively. See ?optim() for details.

\$convergence: An integer code. '0' indicates successful completion (which is always the case for '"SANN"' and '"Brent"'). Possible error codes are:

'1' indicates that the iteration limit 'maxit' had been reached.

'10' indicates degeneracy of the Nelder-Mead simplex.

'51' indicates a warning from the "L-BFGS-B"' method; see component 'message' for further details.

#### МЗВ

'52' indicates an error from the '"L-BFGS-B"' method; see component 'message' for further details. \$message: A character string giving any additional information returned by the optimizer, or 'NULL'.

## Examples

```
testdata = circular::rvonmises(100, mu = circular::circular(pi), kappa = 3)
M3A(testdata)
```

M3B

Model M3B MLE function

## Description

Run Maximum likelihood estimation for model M3B.

## Usage

M3B(data, BadStart, nchains, method, niter)

## Arguments

data	A vector of class 'circular'
BadStart	An integer to replace the log likelihood when starting parameters of the opti- mizer fall outside the preset bounds. This is usually set to a large number. Can also be set to Inf, but will result in an error if a method other than "Nelder-Mead" is chosen. (default = $10^{9}$ )
nchains	A positive integer indicating the number of chains to run. Only the chain with the lowest log likelihood is returned (default = $5$ )
method	A character string indicating the optimizing algorithm to use. Either "BFGS" or "Nelder-Mead" are recommended. See ?optim for more details. (default = "BFGS")
niter	The maximum number of iterations for the optimizing algorithm. Equivalent to the 'maxit' control parameter of the optim() function. See ?optim for more details. (default = 5000)

#### Value

A list with elements (same as for function optim()):

\$par: Vector with the optimized mean angle (mu1), concentration parameter (kappa1), and concentration parameter (kappa2).

\$lik: The negative log likelihood

\$counts: A two-element integer vector giving the number of calls to 'fn' and 'gr' respectively. See ?optim() for details.

\$convergence: An integer code. '0' indicates successful completion (which is always the case for "SANN"' and "Brent"'). Possible error codes are:

'1' indicates that the iteration limit 'maxit' had been reached.

'10' indicates degeneracy of the Nelder-Mead simplex.

'51' indicates a warning from the '"L-BFGS-B"' method; see component 'message' for further details.

'52' indicates an error from the "L-BFGS-B"' method; see component 'message' for further details.

\$message: A character string giving any additional information returned by the optimizer, or 'NULL'.

#### Examples

```
testdata = circular::rvonmises(100, mu = circular::circular(pi), kappa = 3)
M3B(testdata)
```

M4A

#### Model M4A MLE function

## Description

Run Maximum likelihood estimation for model M4A.

#### Usage

M4A(data, BadStart, nchains, method, niter, lambda.min)

## Arguments

data	A vector of class 'circular'
BadStart	An integer to replace the log likelihood when starting parameters of the opti- mizer fall outside the preset bounds. This is usually set to a large number. Can also be set to Inf, but will result in an error if a method other than "Nelder-Mead" is chosen. (default = $10^{9}$ )
nchains	A positive integer indicating the number of chains to run. Only the chain with the lowest log likelihood is returned (default = $5$ )
method	A character string indicating the optimizing algorithm to use. Either "BFGS" or "Nelder-Mead" are recommended. See ?optim for more details. (default = "BFGS")
niter	The maximum number of iterations for the optimizing algorithm. Equivalent to the 'maxit' control parameter of the optim() function. See ?optim for more details. (default = 5000)
lambda.min	The minimum proportional size of the first distribution. Must be between 0 and 1. (default = $0.25$ )

## Value

A list with elements (same as for function optim()):

\$par: Vector with the optimized mean angle (mu1), concentration parameter (kappa1), and proportional size (lambda) of the first distribution.

\$lik: The negative log likelihood

\$counts: A two-element integer vector giving the number of calls to 'fn' and 'gr' respectively. See ?optim() for details.

\$convergence: An integer code. '0' indicates successful completion (which is always the case for "SANN"' and "Brent"'). Possible error codes are:

'1' indicates that the iteration limit 'maxit' had been reached.

'10' indicates degeneracy of the Nelder-Mead simplex.

'51' indicates a warning from the "L-BFGS-B"' method; see component 'message' for further details.

'52' indicates an error from the "L-BFGS-B"' method; see component 'message' for further details.

\$message: A character string giving any additional information returned by the optimizer, or 'NULL'.

#### Examples

```
testdata = circular::rvonmises(100, mu = circular::circular(pi), kappa = 3)
M4A(testdata)
```

M4B

Model M4B MLE function

## Description

Run Maximum likelihood estimation for model M4B.

## Usage

M4B(data, BadStart, nchains, method, niter, lambda.min)

#### Arguments

data	A vector of class 'circular'
BadStart	An integer to replace the log likelihood when starting parameters of the opti- mizer fall outside the preset bounds. This is usually set to a large number. Can also be set to Inf, but will result in an error if a method other than "Nelder-Mead" is chosen. (default = 10^9)
nchains	A positive integer indicating the number of chains to run. Only the chain with the lowest log likelihood is returned (default = $5$ )
method	A character string indicating the optimizing algorithm to use. Either "BFGS" or "Nelder-Mead" are recommended. See ?optim for more details. (default = "BFGS")

niter	The maximum number of iterations for the optimizing algorithm. Equivalent to the 'maxit' control parameter of the optim() function. See ?optim for more details. (default = 5000)
lambda.min	The minimum proportional size of the first distribution. Must be between 0 and 1. (default = $0.25$ )

## Value

A list with elements (same as for function optim()):

\$par: Vector with the optimized mean angle (mu1), concentration parameter (kappa1), concentration parameter (kappa2), and proportional size (lambda) of the first distribution.

\$lik: The negative log likelihood

\$counts: A two-element integer vector giving the number of calls to 'fn' and 'gr' respectively. See ?optim() for details.

\$convergence: An integer code. '0' indicates successful completion (which is always the case for "SANN" and "Brent"). Possible error codes are:

'1' indicates that the iteration limit 'maxit' had been reached.

'10' indicates degeneracy of the Nelder-Mead simplex.

'51' indicates a warning from the "L-BFGS-B"' method; see component 'message' for further details.

'52' indicates an error from the "L-BFGS-B"' method; see component 'message' for further details.

\$message: A character string giving any additional information returned by the optimizer, or 'NULL'.

#### Examples

```
testdata = circular::rvonmises(100, mu = circular::circular(pi), kappa = 3)
M4B(testdata)
```

M5A

Model M5A MLE function

#### Description

Run Maximum likelihood estimation for model M5A.

## Usage

```
M5A(data, BadStart, nchains, method, niter, lambda.min, q.diff)
```

## M5A

## Arguments

data	A vector of class 'circular'
BadStart	An integer to replace the log likelihood when starting parameters of the opti- mizer fall outside the preset bounds. This is usually set to a large number. Can also be set to Inf, but will result in an error if a method other than "Nelder-Mead" is chosen. (default = $10^{9}$ )
nchains	A positive integer indicating the number of chains to run. Only the chain with the lowest log likelihood is returned (default = $5$ )
method	A character string indicating the optimizing algorithm to use. Either "BFGS" or "Nelder-Mead" are recommended. See ?optim for more details. (default = "BFGS")
niter	The maximum number of iterations for the optimizing algorithm. Equivalent to the 'maxit' control parameter of the optim() function. See ?optim for more details. (default = 5000)
lambda.min	The minimum proportional size of the first distribution. Must be between 0 and 1. (default = $0.25$ )
q.diff	The minimum difference (in radians) in preferred direction for bimodal models. Must be set between 0 and pi. (default = $pi/4$ )

#### Value

A list with elements (same as for function optim()):

\$par: Vector with the optimized mean angle (mu1), concentration parameter (kappa1), mean angle (mu2), and proportional size (lambda) of the first distribution.

\$lik: The negative log likelihood

\$counts: A two-element integer vector giving the number of calls to 'fn' and 'gr' respectively. See ?optim() for details.

\$convergence: An integer code. '0' indicates successful completion (which is always the case for "SANN"' and "Brent"'). Possible error codes are:

'1' indicates that the iteration limit 'maxit' had been reached.

'10' indicates degeneracy of the Nelder-Mead simplex.

'51' indicates a warning from the "L-BFGS-B"' method; see component 'message' for further details.

'52' indicates an error from the "L-BFGS-B"' method; see component 'message' for further details.

\$message: A character string giving any additional information returned by the optimizer, or 'NULL'.

## Examples

```
testdata = circular::rvonmises(100, mu = circular::circular(pi), kappa = 3)
M5A(testdata)
```

## Description

Run Maximum likelihood estimation for model M5B.

#### Usage

M5B(data, BadStart, nchains, method, niter, lambda.min, q.diff)

#### Arguments

data	A vector of class 'circular'
BadStart	An integer to replace the log likelihood when starting parameters of the opti- mizer fall outside the preset bounds. This is usually set to a large number. Can also be set to Inf, but will result in an error if a method other than "Nelder-Mead" is chosen. (default = 10^9)
nchains	A positive integer indicating the number of chains to run. Only the chain with the lowest log likelihood is returned (default = $5$ )
method	A character string indicating the optimizing algorithm to use. Either "BFGS" or "Nelder-Mead" are recommended. See ?optim for more details. (default = "BFGS")
niter	The maximum number of iterations for the optimizing algorithm. Equivalent to the 'maxit' control parameter of the optim() function. See ?optim for more details. (default = 5000)
lambda.min	The minimum proportional size of the first distribution. Must be between 0 and 1. (default = $0.25$ )
q.diff	The minimum difference (in radians) in preferred direction for bimodal models. Must be set between 0 and pi. (default = $pi/4$ )

#### Value

A list with elements (same as for function optim()):

\$par: Vector with the optimized mean angle (mu1), concentration parameter (kappa1), mean angle (mu2), concentration parameter (kappa2), and proportional size (lambda) of the first distribution.

\$lik: The negative log likelihood

\$counts: A two-element integer vector giving the number of calls to 'fn' and 'gr' respectively. See ?optim() for details.

\$convergence: An integer code. '0' indicates successful completion (which is always the case for "SANN"' and "Brent"'). Possible error codes are:

'1' indicates that the iteration limit 'maxit' had been reached.

'10' indicates degeneracy of the Nelder-Mead simplex.

M5B

## plot\_circMLE

'51' indicates a warning from the "L-BFGS-B"' method; see component 'message' for further details.

'52' indicates an error from the '"L-BFGS-B"' method; see component 'message' for further details.

\$message: A character string giving any additional information returned by the optimizer, or 'NULL'.

## Examples

```
testdata = circular::rvonmises(100, mu = circular::circular(pi), kappa = 3)
M5B(testdata)
```

plot\_circMLE plot\_circMLE

## Description

Plot the observed data and specific model from circ\_mle output

#### Usage

plot\_circMLE(data, table, model, bins, shrink, col, lwd, lty)

## Arguments

data	A vector of class 'circular'
table	A list containing the output from the circ_mle function
model	The name of the model to be plotted. (default = first model of "table")
bins	The number of bins for the circular histogram. (defualt = $18$ )
shrink	The value by which to shrink the size of the plotted circle. Larger numbers shrink the circle, smaller numbers expand the circle. (default = $1.5$ )
col	Vector of colors used for plotting, up to four colors can be specified. The order is: histogram color, mean vector color, model density color, predicted mean direction(s) color(s). (default = c("grey", "red", "black", "black"))
lwd	Vector of line weights used for plotting, up to 3 weights can be specified. The order is: mean vector weight, model density weight, predicted mean direction(s) weight(s). (default = $c(2, 2, 2)$ )
lty	Vector of line weights used for plotting, up to 3 weights can be specified. The order is: mean vector weight, model density weight, predicted mean direction(s) weight(s). (default = c("solid", "dashed", "dashed"))

#### Value

A plot is returned to the default image device.

## Note

In the resulting plot, the observed vector length is equal to rho (vector strength). However, the predicted direction(s) from the model are drawn with vector length of 1. Also, if the model "M1" is selected, by definition no predicted direction is plotted.

#### Examples

```
testdata = circular::rvonmises(100, mu = circular::circular(pi), kappa = 3)
out = circ_mle(testdata)
plot_circMLE(testdata, out)
plot_circMLE(testdata, out, model = "M4A")
```

pycke\_test Pycke test function

## Description

Perform the Pycke test.

#### Usage

pycke\_test(data, iter = 9999)

#### Arguments

data	A vector of class 'circular', or numeric vector of angles measured in radians
iter	The number of bootstrap replicates to perform in order to estimate the p-value
	of the test. (default = $9999$ )

#### Value

A numeric vector of the test statistic (T) and associated p-value

#### Examples

```
testdata = circular::rvonmises(20, mu = circular::circular(pi), kappa = 3)
pycke_test(testdata, iter = 999)
```

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