

# Package ‘BNSL’

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**Title** Bayesian Network Structure Learning

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**Author** Joe Suzuki and Jun Kawahara

**Maintainer** Joe Suzuki <j-suzuki@sigmath.es.osaka-u.ac.jp>

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**Description** From a given data frame, this package learns its Bayesian network structure based on a selected score.

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 BNSL-package

*Bayesian Network Structure Learning*


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### Description

From a given dataframe, this package learns a Bayesian network structure based on a selected score.

### Details

Currently, this package estimates of mutual information and conditional mutual information, and combines them to construct either a Bayesian network or an undirected forest. Any undirected forest can be a Bayesian network by adding appropriate directions.

### Author(s)

Joe Suzuki and Jun Kawahara

Maintainer: Joe Suzuki <j-suzuki@sigmath.es.osaka-u.ac.jp>

### References

[1] Suzuki, J., "A theoretical analysis of the BDeu scores in Bayesian network structure learning", *Behaviormetrika*, 2017. [2] Suzuki, J., "A novel Chow-Liu algorithm and its application to gene differential analysis", *International Journal of Approximate Reasoning*, 2017. [3] Suzuki, J., "Efficient Bayesian network structure learning for maximizing the posterior probability", *Next-Generation Computing*, 2017. [4] Suzuki, J., "An estimator of mutual information and its application to independence testing", *Entropy*, Vol.18, No.4, 2016. [5] Suzuki, J., "Consistency of learning Bayesian network structures with continuous variables: An information theoretic approach". *Entropy*, Vol.17, No.8, 5752-5770, 2015. [6] Suzuki, J., "Learning Bayesian network structures when discrete and continuous variables are present. In *Lecture Note on Artificial Intelligence, the sixth European workshop on Probabilistic Graphical Models*, Vol. 8754, pp. 471-486, Utrecht, Netherlands, Sept. 2014. Springer-Verlag. [7] Suzuki, J., "The Bayesian Chow-Liu algorithms", In the sixth European workshop on Probabilistic Graphical Models, pp. 315-322, Granada, Spain, Sept. 2012. [8] Suzuki, J. and Kawahara, J., "Branch and Bound for Regular Bayesian Network Structure learning", *Uncertainty in Artificial Intelligence*, pages 212-221, Sydney, Australia, August 2017. [9] Suzuki, J. "Forest Learning from Data and its Universal Coding", *IEEE Transactions on Information Theory*, Dec. 2018. January 2017.

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 bns1

*Bayesian Network Structure Learning*


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### Description

The function outputs the Bayesian network structure given a dataset based on an assumed criterion.

**Usage**

```
bns1(df, tw = 0, proc = 1, s=0, n=0, ss=1)
```

**Arguments**

df	a dataframe.
tw	the upper limit of the parent set.
proc	the criterion based on which the BNSL solution is sought. proc=1,2, and 3 indicates that the structure learning is based on Jeffreys [1], MDL [2,3], and BDeu [3]
s	The value computed when obtaining the bound.
n	The number of samples.
ss	The BDeu parameter.

**Value**

The Bayesian network structure in the bn class of bnlearn.

**Author(s)**

Joe Suzuki and Jun Kawahara

**References**

[1] Suzuki, J. "An Efficient Bayesian Network Structure Learning Strategy", New Generation Computing, December 2016. [2] Suzuki, J. "A construction of Bayesian networks from databases based on an MDL principle", Uncertainty in Artificial Intelligence, pages 266-273, Washington D.C. July, 1993. [3] Suzuki, J. "Learning Bayesian Belief Networks Based on the Minimum Description Length Principle: An Efficient Algorithm Using the B & B Technique", International Conference on Machine Learning, Bali, Italy, July 1996" [4] Suzuki, J. "A Theoretical Analysis of the BDeu Scores in Bayesian Network Structure Learning", Behaviormetrika 1(1):1-20, [5] Suzuki, J. and Kawahara, J., "Branch and Bound for Regular Bayesian Network Structure learning", Uncertainty in Artificial Intelligence, pages 212-221, Sydney, Australia, August 2017. [6] Suzuki, J. "Forest Learning from Data and its Universal Coding", IEEE Transactions on Information Theory, Dec. 2018. January 2017.

**See Also**

parent

**Examples**

```
library(bnlearn)
bns1(asia)
```

bns1\_p

*Bayesian Network Structure Learning***Description**

The function outputs the Bayesian network structure given a dataset based on an assumed criterion.

**Usage**

```
bns1_p(df, ps1, tw = 0, proc = 1, s=0, n=0, ss=1)
```

**Arguments**

df	a dataframe.
ps1	the list of parent sets.
tw	the upper limit of the parent set.
proc	the criterion based on which the BNSL solution is sought. proc=1,2, and 3 indicates that the structure learning is based on Jeffreys [1], MDL [2,3], and BDeu [3]
s	The value computed when obtaining the bound.
n	The number of samples.
ss	The BDeu parameter.

**Value**

The Bayesian network structure in the bn class of bnlearn.

**Author(s)**

Joe Suzuki and Jun Kawahara

**References**

[1] Suzuki, J. "An Efficient Bayesian Network Structure Learning Strategy", New Generation Computing, December 2016. [2] Suzuki, J. "A construction of Bayesian networks from databases based on an MDL principle", Uncertainty in Artificial Intelligence, pages 266-273, Washington D.C. July, 1993. [3] Suzuki, J. "Learning Bayesian Belief Networks Based on the Minimum Description Length Principle: An Efficient Algorithm Using the B & B Technique", International Conference on Machine Learning, Bali, Italy, July 1996" [4] Suzuki, J. "A Theoretical Analysis of the BDeu Scores in Bayesian Network Structure Learning", Behaviormetrika 1(1):1-20, January 2017.

**See Also**

parent

## Examples

```
library(bnlearn)
p0 <- parent.set(lizards, 0)
p1 <- parent.set(lizards, 1)
p2 <- parent.set(lizards, 2)
bns1_p(lizards, list(p0, p1, p2))
```

cmi

*Bayesian Estimation of Conditional Mutual Information*

## Description

A standard estimator of conditional mutual information calculates the maximal likelihood value. However, the estimator takes positive values even the pair follows a distribution of two independent variables. On the other hand, the estimator in this package detects conditional independence as well as consistently estimates the true conditional mutual information value as the length grows based on Jeffrey's prior, Bayesian Dirichlet equivalent uniform (BDeu [1]), and the MDL principle. It also estimates the conditional mutual information value even when one of the pair is continuous (see [2]).

## Usage

```
cmi(x, y, z, proc=0L)
```

## Arguments

x	a numeric vector.
y	a numeric vector.
z	a numeric vector. x, y and z should have an equal length.
proc	the estimation is based on Jeffrey's prior, the MDL principle, and BDeu for proc=0,1,2, respectively. If the argument proc is missing, proc=0 (Jeffreys') is assumed.

## Value

the estimation of conditional mutual information between the two numeric vectors based on the selected criterion, where the natural logarithm base is assumed.

## Author(s)

Joe Suzuki and Jun Kawahara

## References

[1] Suzuki, J., "A theoretical analysis of the BDeu scores in Bayesian network structure learning", Behaviormetrika, 2017. [2] Suzuki, J., "An estimator of mutual information and its application to independence testing", Entropy, Vol.18, No.4, 2016. [3] Suzuki, J. "The Bayesian Chow-Liu algorithms", In the sixth European workshop on Probabilistic Graphical Models, pp. 315-322, Granada, Spain, Sept.2012.

**See Also**

cmi

**Examples**

```
n=100

x=c(rbinom(n,1,0.2), rbinom(n,1,0.8))
y=c(rbinom(n,1,0.8), rbinom(n,1,0.2))
z=c(rep(1,n),rep(0,n))
cmi(x,y,z,proc=0); cmi(x,y,z,1); cmi(x,y,z,2)

x=c(rbinom(n,1,0.2), rbinom(n,1,0.8))
u=rbinom(2*n,1,0.1)
y=(x+u)
z=c(rep(1,n),rep(0,n))
cmi(x,y,z); cmi(x,y,z,proc=1); cmi(x,y,z,2)
```

---

FFtable

*A faster version of fftable*


---

**Description**

The same procedure as fftable prepared by the R language. The program is written using Rcpp.

**Usage**

```
FFtable(df)
```

**Arguments**

df                      a dataframe.

**Value**

a frequency table of the last column based on the states that are determined by the other columns.

**Author(s)**

Joe Suzuki and Jun Kawahara

**See Also**

fftable

**Examples**

```
library(bnlearn)
FFtable(asia)
```

---

`kruskal`*Given a weight matrix, generate its maximum weight forest*

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**Description**

The function lists the edges of an forest generated by Kruskal's algorithm given its weight matrix in which each weight should be symmetric but may be negative. The forest is a spanning tree if the elements of the matrix take positive values.

**Usage**

```
kruskal(W)
```

**Arguments**

`W` a matrix.

**Value**

A matrix object of size  $n \times 2$  for matrix size  $n \times n$  in which each row expresses an edge when the vertexes are expressed by 1 through  $n$ .

**Author(s)**

Joe Suzuki and Jun Kawahara

**References**

[1] Suzuki. J. "The Bayesian Chow-Liu algorithms", In the sixth European workshop on Probabilistic Graphical Models, pp. 315-322, Granada, Spain, Sept.2012.

**Examples**

```
library(igraph)
library(bnlearn)
df=asia
mi.mat=mi_matrix(df)
edge.list=kruskal(mi.mat)
edge.list
g=graph_from_edgelist(edge.list, directed=FALSE)
V(g)$label=colnames(df)
plot(g)
```

mi

*Bayesian Estimation of Mutual Information***Description**

A standard estimator of mutual information calculates the maximal likelihood value. However, the estimator takes positive values even the pair follows a distribution of two independent variables. On the other hand, the estimator in this package detects independence as well as consistently estimates the true mutual information value as the length grows based on Jeffrey's prior, Bayesian Dirichlet equivalent uniform (BDeu [1]), and the MDL principle. It also estimates the mutual information value even when one of the pair is continuous (see [2]).

**Usage**

```
mi(x, y, proc=0)
```

**Arguments**

x	a numeric vector.
y	a numeric vector. x and y should have a equal length.
proc	the estimation is based on Jeffrey's prior, the MDL principle, and BDeu for proc=0,1,2, respectively. If one of the two is continuous, proc=10 should be chosen. If the argument proc is missing, proc=0 (Jeffreys') is assumed.

**Value**

the estimation of mutual information between the two numeric vectors based on the selected criterion, where the natural logarithm base is assumed.

**Author(s)**

Joe Suzuki and Jun Kawahara

**References**

[1] Suzuki, J., "A theoretical analysis of the BDeu scores in Bayesian network structure learning", Behaviormetrika, 2017. [2] Suzuki, J., "An estimator of mutual information and its application to independence testing", Entropy, Vol.18, No.4, 2016. [3] Suzuki. J. "The Bayesian Chow-Liu algorithms", In the sixth European workshop on Probabilistic Graphical Models, pp. 315-322, Granada, Spain, Sept.2012.

**See Also**

cmi



**Examples**

```

n=100

x=rbinom(n,1,0.5); y=rbinom(n,1,0.5); mi(x,y)

z=rbinom(n,1,0.1); y=(x+z)

mi(x,y); mi(x,y,proc=1); mi(x,y,2)

x=rnorm(n); y=rnorm(n); mi(x,y,proc=10)

x=rnorm(n); z=rnorm(n); y=0.9*x+sqrt(1-0.9^2)*z; mi(x,y,proc=10)

```

mi\_matrix

*Generating its Mutual Information Estimations Matrix***Description**

The estimators in this package detect independence as well as consistently estimates the true conditional mutual information value as the length grows based on Jeffrey's prior, Bayesian Dirichlet equivalent uniform (BDeu [1]), and the MDL principle. It also estimates the conditional mutual information value even when one of the pair is continuous (see [2]). Given a data frame each column of which may be either discrete or continuous, this function generates its mutual information estimation matrix.

**Usage**

```
mi_matrix(df, proc=0)
```

**Arguments**

df	a data frame.
proc	given two discrete vectors of equal length, the function estimates the mutual information based on Jeffrey's prior, the MDL principle, and BDeu for proc=0,1,2, respectively. If one of the columns is continuous, proc=10 should be chosen. If the argument proc is missing, proc=0 (Jeffreys') is assumed.

**Value**

the estimation of mutual information between the two numeric vectors based on the selected criterion, where the natural logarithm base is assumed.

**Author(s)**

Joe Suzuki and Jun Kawahara

## References

[1] Suzuki, J., "A theoretical analysis of the BDeu scores in Bayesian network structure learning", Behaviormetrika, 2017. [2] Suzuki, J., "An estimator of mutual information and its application to independence testing", Entropy, Vol.18, No.4, 2016. [3] Suzuki, J., "A novel Chow-Liu algorithm and its application to gene differential analysis", International Journal of Approximate Reasoning, Vol. 80, 2017.

## See Also

mi

## Examples

```
library(bnlearn)
mi_matrix(asia)
mi_matrix(asia,proc=1)
mi_matrix(asia,proc=2)
mi_matrix(asia,proc=3)
```

---

parent.set

*Parent Set*

---

## Description

This function estimates a parent set of  $h$  in each subset  $w$  as follows: Suppose we are given a subset  $w$  of the  $p-1$  variables excluding  $h$ , where  $p$  is the number of columns in  $df$ . Then, a score is defined for each subset  $w$ , where the score expresses how well the subset is likely to be the true parent set of  $h$  in  $w$ . Currently, a Bayesian score (Jeffreys' prior) is applied. This function computes the maximum score  $z$  and its subset  $y$  of  $w$ . This function computes  $y$  and  $z$  for all  $w$ , where  $w$  and  $y$  are expressed by binary sequences of length  $p$ , respectively. When the computation is heavy, it can be reduced by specifying the maximum size of  $w$ . If  $tw$  is zero (default), the  $tw$  value is set to  $p-1$ . Otherwise, the  $tw$  value expresses the maximum size.

## Usage

```
parent.set(df, h, tw=0, proc=1)
```

## Arguments

<code>df</code>	a data frame.
<code>h</code>	an integer from 0 to $p-1$ , where $p$ is the number of columns in $df$ .
<code>tw</code>	an integer from 0 to $p-1$ , where $p$ is the number of columns in $df$ .
<code>proc</code>	the parent sets are estimated based on Jeffreys' (proc=0,1) [1], MDL (proc=2) [2,3], and BDeu (proc=3) [4].

**Value**

the data frame in which each row consists of the triples (w,y,z): w is a subset of the p-1 variables excluding h; y is the parent set for w; and z is the score of the parent set.

**Author(s)**

Joe Suzuki and Jun Kawahara

**References**

[1] Suzuki, J., "An Efficient Bayesian Network Structure Learning Strategy", New Generation Computing, December 2016. [2] Suzuki, A., "Construction of Bayesian Networks from Databases Based on an MDL Principle", Proceedings of the Ninth Annual Conference on Uncertainty in Artificial Intelligence, The Catholic University of America, Providence, Washington, DC, USA, July 9-11, 1993. [3] Suzuki, J., "Learning Bayesian Belief Networks Based on the Minimum Description Length Principle: An Efficient Algorithm Using the B & B Technique.", Proceedings of the Thirteenth International Conference (ICML '96), Bari, Italy, July 3-6, 1996. [4] Suzuki, J., "A theoretical analysis of the BDeu scores in Bayesian network structure learning", Behaviormetrika, 2017.

**See Also**

cmi

**Examples**

```
library(bnlearn)
df=asia
parent.set(df,7)
parent.set(df,7,1)
parent.set(df,7,2)
```

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